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AN ANALYTICAL STUDY OF AIRPLANE-AUTOPILOT RESPONSE TO ATMOSPHERIC TURBULENCE

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16 Abstract

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AN ANALYTICAL STUDY OF AIRPLANE-AUTOPILOT RESPONSE TO ATMOSPHERIC TURBULENCE

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SUMMARY

An analytical study has shown that, with proper selection of feedback gains, an automatic control system can reduce excursions in altitude of a jet transport flying in turbulence without increasing structural loads. The control system uses feedback of attitude-angle and pitch-rate signals to the elevator to provide attitude hold and uses feedback of altitude and altitude-rate signals to the throttle to provide altitude hold. Without the altitude-hold control, altitude excursions of the airplane are sufficient, in mild turbulence, to cause the airplane to exceed its allowable altitude corridor. The altitude-hold mode is effective in reducing altitude excursions by about 50 percent. In extremely severe turbulence the airplane exceeds its limit normal-load factor with the control system in operation.

INTRODUCTION

The use of autopilots to reduce the response of airplanes to turbulence has been a controversial subject for many years. Earlier, it was thought that autopilots increase airplane response to turbulence and pilots were instructed to turn off the autopilot in rough air. Late in 1968, based on an extensive simulation study, it was recommended that an operational autopilot ("turbulence flight director") be developed for use during flight through turbulence (ref. 1). Recently, the use of autopilots during flight in turbulence has become more widely accepted. However, there have been only a few studies made by using various combinations of feedback quantities that give quantitative responses to both vertical and horizontal gusts in autopilot-controlled flight. Some exploratory results, obtained by using different analyses, are presented in references 2 and 3.

The present study extends the study of reference 2 by consideration of more than one flight condition and by inclusion of filters and servos in the automatic control system. The objective is to determine what controls and what feedback quantities can be used to reduce excursions in speed and altitude from the desired trim condition without increasing structural loads on the airplane.

Responses to atmospheric turbulence of a medium jet transport airplane are calculated for two altitudes and for several Mach numbers. The basic airplane and the airplane operating with each of three variations of the automatic control system are studied. The calculated variables of motion are the root-mean-square (rms) values of airspeed, angle of attack, attitude angle, altitude, pitch rate, and normal-load factor. The responses are calculated for both vertical and horizontal components of turbulence.

SYMBOLS

$[B(\omega')]$ matrix defined by equation (9)

C_D drag coefficient, $\frac{\text{Drag}}{qS}$

C_L lift coefficient, $\frac{\text{Lift}}{qS} = \frac{\bar{C}_L qS - C_T \sin \alpha qS}{qS}$

$\bar{C}_L = \frac{mg}{qS}$

C_m pitching-moment coefficient, $\frac{\text{Pitching moment}}{qS\bar{c}}$

C_T thrust coefficient, $\frac{\text{Thrust}}{qS}$

$\bar{c}$ wing mean aerodynamic chord, meters

F_X force along X-axis

F_Z force along Z-axis

$f_1(U),f_2(U),f_3(U)$ functions of gust velocity

g gravitational constant, meters/second^2 (1g = 9.807 m/sec^2)

$[H(\omega')]$ matrix of frequency-response functions

h altitude, meters

h_c command altitude, meters

K_θ,K_q,K_t automatic-control gains (see fig. 2)

k_Y	airplane radius of gyration, meters
L	longitudinal scale of turbulence, meters
M	Mach number
M_Y	moment about Y-axis
m	mass of airplane, kilograms
n	normal-load factor, $\frac{\text{Normal acceleration}}{g}$
q	free-stream dynamic pressure, newtons/meter^2
S	wing area, meters^2
s	Laplace variable, 1/second
t	time, seconds
t'	nondimensional time, $\frac{tV}{\bar{c}}$
U	velocity of local air mass (gust velocity), meters/second, $U = \sqrt{u^2 + v^2 + w^2}$
u	longitudinal component of U, meters/second
V	airspeed, meters/second or knots
v	lateral component of U, meters/second
w	vertical component of U, meters/second
α	angle of attack, radians
β	change of air density with altitude, 1/meter, $-\frac{1}{\rho}\frac{d\rho}{dh}$
δ_e	elevator deflection angle, radians
$\delta_{e,c}$	command elevator deflection angle, radians

δ_t throttle deflection angle, radians

θ attitude angle, radians

θ_c command attitude angle, radians

λ nondimensional Laplace variable, $\frac{\bar{c}}{V}s$

μ nondimensional mass, $\frac{m}{\rho S\bar{c}}$

ρ air density, kilograms/meter^3

σ_h rms (root-mean-square) altitude, meters

σ_n rms normal-load factor

σ_q *rms pitch rate, radians/second*

σ_u rms horizontal component of gust velocity, meters/second

σ_V rms airspeed, meters/second or knots

σ_w rms vertical component of gust velocity, meters/second

σ_α rms angle of attack, radians

σ_θ rms attitude angle, radians

$\Phi_u(\omega')$ power spectral density of horizontal component of gust velocity, meters^2/second^2

$\Phi_w(\omega')$ power spectral density of vertical component of gust velocity, meters^2/second^2

Ω spatial frequency, $\frac{\omega'}{\bar{c}}$, radians

ω circular frequency, radians/second

ω' reduced frequency, $\frac{\omega\bar{c}}{V}$, radians

Notation:

˙ (Dot) first derivative with respect to time t

¨ (Double dot) second derivative with respect to time t

' (Prime) first derivative with respect to nondimensional time t'; for example,
$$\alpha' = \frac{d\alpha}{dt'}$$

'' (Double prime) second derivative with respect to nondimensional time t'

Δ perturbation quantity

Derivatives are defined as follows:

$$C_{D_V} = V\frac{\partial C_D}{\partial V} \qquad C_{m_\alpha} = \frac{\partial C_m}{\partial \alpha}$$

$$C_{D_\alpha} = \frac{\partial C_D}{\partial \alpha} \qquad C_{m_{\alpha'}} = \frac{\partial C_m}{\partial \alpha'}$$

$$C_{L_\alpha} = \frac{\partial C_L}{\partial \alpha} \qquad C_{m_{\delta_e}} = \frac{\partial C_m}{\partial \delta_e}$$

$$C_{L_{\alpha'}} = \frac{\partial C_L}{\partial \alpha'} \qquad C_{m_{\theta'}} = \frac{\partial C_m}{\partial \theta'}$$

$$C_{L_{\delta_e}} = \frac{\partial C_L}{\partial \delta_e} \qquad C_{T_V} = V\frac{\partial C_T}{\partial V}$$

$$C_{L_{\theta'}} = \frac{\partial C_L}{\partial \theta'} \qquad C_{T_{\delta_t}} = \frac{\partial C_T}{\partial \delta_t}$$

$$C_{m_V} = V\frac{\partial C_m}{\partial V} \qquad C_{T_\rho} = \rho\frac{\partial C_T}{\partial \rho}$$

THEORY

Equations of Longitudinal Motion

The derivation of the equations of longitudinal motion of a power-fixed airplane is well documented. (See ref. 4.) Since the equations used in the present study have terms involving the variation of thrust with altitude change and because the derivation of equations with these terms is not well documented, a derivation is presented in the appendix.

The equations of airplane motion are linear differential equations in perturbation variables. The mass of a rigid airplane is assumed to be concentrated at the origin of a system of moving wind axes as depicted in figure 1. The X-axis is always alined with the velocity vector. Although the airplane may rotate with respect to the moving axes, it is assumed that changes in the moment of inertia I_Y are negligible. Finally, the longitudinal motions are assumed to be uncoupled from lateral motions. In accord with these assumptions, the equations of longitudinal motion in Laplace notation are written as follows:

$$\left.\begin{aligned}
&\left(2\mu\lambda - C_{T_V}\cos\alpha + C_{D_V}\right)\frac{\Delta V}{V} - \left(C_L - C_{D_\alpha}\right)\Delta\alpha + \overline{C}_L\,\Delta\theta + \bar{c}\beta\left(C_{T_\rho} + C_T\right)\cos\alpha\,\frac{\Delta h}{\bar{c}} \\
&\qquad = -C_{T_{\delta_t}}\delta_t + f_1(U) \\
&\left(C_{T_V}\sin\alpha + 2\overline{C}_L\right)\frac{\Delta V}{V} + \left[C_T\cos\alpha + C_{L_\alpha} + \left(C_{L_{\alpha'}} + 2\mu\right)\lambda\right]\Delta\alpha + \left(C_{L_{\theta'}} - 2\mu\right)\lambda\,\Delta\theta \\
&\quad - \bar{c}\beta\left(C_{T_\rho} + C_T\right)\sin\alpha\,\frac{\Delta h}{\bar{c}} = -C_{L_{\delta_e}}\delta_e + f_2(U) \\
&C_{m_V}\frac{\Delta V}{V} + \left(C_{m_\alpha} + C_{m_{\alpha'}}\lambda\right)\Delta\alpha + \left(C_{m_{\theta'}}\lambda - 2\mu\frac{k_Y^2}{\bar{c}^2}\lambda^2\right)\Delta\theta = -C_{m_{\delta_e}}\delta_e + f_3(U) \\
&\Delta\alpha - \Delta\theta + \lambda\frac{\Delta h}{\bar{c}} = 0
\end{aligned}\right\} \tag{1}$$

The desired equilibrium conditions are

$$\left.\begin{aligned}
&C_T\cos\alpha = C_D \\
&C_L = -C_T\sin\alpha + \frac{2mg}{\rho SV^2} \\
&C_m = 0 \\
&\theta = \alpha
\end{aligned}\right\} \tag{2}$$

Equations (1) are made nondimensional in the following ways:

(1) The force equations are divided by $\frac{\rho}{2}SV^2$

(2) The moment equation is divided by $\frac{\rho}{2}S\bar{c}V^2$

(3) Time derivatives are taken with respect to nondimensional time $t' = \frac{tV}{\bar{c}}$

The coefficients of the perturbation variables in equations (1) are evaluated at equilibrium conditions. Control inputs are throttle deflection angle δ_t and elevator deflection angle δ_e. These inputs are given as transfer functions in the next section as equations (3) and (4). The functions of gust velocity $f_1(U)$, $f_2(U)$, and $f_3(U)$ represent the turbulence input.

Equations for Automatic Control System

Two variations of the automatic control system are to be considered both separately and combined. One uses altitude and rate-of-climb signals to produce throttle changes. The purpose of this "autothrottle" is to maintain a desired altitude. The other uses attitude-angle and pitch-rate signals to produce elevator deflections. The purpose of this "pitch autopilot" is to maintain a desired attitude angle. A block diagram of the automatic control system is shown in figure 2 The pitch autopilot, with gains $K_\theta = 2.36$ and $K_q = 3.2$, is the attitude-hold mode of a conventional autopilot.

The transfer function for the pitch autopilot, relating elevator deflection to attitude angle, is given in Laplace transform notation as follows:

$$\frac{\delta_e}{\Delta\theta}(\lambda) = \left[K_\theta + \underbrace{\frac{18.5\frac{V}{\bar{c}}\lambda}{\left(\frac{V}{\bar{c}}\lambda + 1\right)\left(\frac{V}{\bar{c}}\lambda + 18.5\right)}}_{\text{Band-pass filter}}\frac{V}{\bar{c}}\lambda K_q\right]\underbrace{\frac{144}{\left(\frac{V}{\bar{c}}\lambda + 12\right)^2}}_{\text{Servo}} \tag{3}$$

The gains K_θ and K_q are adjustable.

The transfer function for the autothrottle, relating change of thrust coefficient with altitude, is given in Laplace notation as follows:

$$\frac{\delta_t}{\Delta h/\bar{c}}\frac{C_T}{\delta_t}(\lambda) = \frac{C_T}{\Delta h/\bar{c}}(\lambda) = -0.004\left(\frac{V}{\bar{c}}\lambda + 0.025\right)K_t\bar{c}\,\underbrace{\frac{1}{\left(4\frac{V}{\bar{c}}\lambda + 1\right)\left(4.4\frac{V}{\bar{c}}\lambda + 1\right)}}_{\text{Time lag}} \tag{4}$$

The gain K_t is adjustable. This autothrottle is essentially the one used in reference 2.

Analytical Representation of Atmospheric Turbulence

Discussion of the random properties of atmospheric turbulence and the derivation of analytical representations are given in reference 5. The Von Kármán one-dimensional

power-spectral-density representations are, for the horizontal component of gust velocity,

$$\Phi_u(\omega') = \sigma_u^2 \frac{2L}{\pi\bar{c}} \frac{1}{\left[1 + \left(1.339 \frac{L\omega'}{\bar{c}}\right)^2\right]^{5/6}} \tag{5}$$

and, for the vertical component of gust velocity,

$$\Phi_w(\omega') = \sigma_w^2 \frac{L}{\pi\bar{c}} \frac{1 + \frac{8}{3}\left(1.339 \frac{L\omega'}{\bar{c}}\right)^2}{\left[1 + \left(1.339 \frac{L\omega'}{\bar{c}}\right)^2\right]^{11/6}} \tag{6}$$

The reduced frequency ω' is related to spatial frequency Ω and circular frequency ω by

$$\Omega = \frac{\omega'}{\bar{c}}$$

and

$$\omega = \frac{V}{\bar{c}} \omega'$$

The longitudinal scale of turbulence L may be interpreted, physically, in various ways (see appendix B of ref. 5), and its value for atmospheric turbulence depends on altitude. One physical interpretation is that L is a rough measure of the longest distance by which two points in a turbulence field may be separated before the correlation function between the velocities becomes zero. The scale of turbulence is assumed to be 0.762 km for altitudes of 6.096 km and 10.668 km. Mean-square values of horizontal and vertical components of gust velocity are denoted by σ_u^2 and σ_w^2, respectively. The power spectra $\Phi_u(\omega')$ and $\Phi_w(\omega')$ are plotted in figure 3. Use of statistical properties of atmospheric turbulence requires the application of random-process theory to obtain the statistical properties of the airplane response.

Random-Process Theory

The necessary mathematical development of random-process theory may be obtained from reference 6. An important result relates the power spectral densities of the input and the output of a linear system. The relationship may be stated as

$$\Phi_o(\omega') = \left|H(\omega')\right|^2 \Phi_i(\omega') \tag{7}$$

where $[H(\omega')]$ is the frequency-response function of the linear system to a sinusoidal input. The function $\Phi_i(\omega')$ is the power spectral density of the input; and the function $\Phi_o(\omega')$ is the power spectral density of the output. Application of equation (7) to problems involving the response of airplanes to atmospheric turbulence is discussed thoroughly in references 5, 7, and 8.

For the problem considered, there is a frequency-response function for each of the variables $\Delta V/V$, $\Delta\alpha$, $\Delta\theta$, and $\Delta h/\bar{c}$. These frequency-response functions are obtained from the equations of motion (see eqs. (1)) written in matrix form with the substitution of $\lambda = i\omega'$. The resulting matrix of coefficients, which contain the control transfer functions, is represented as the matrix $[Z(i\omega')]$. The coefficients in the gust-velocity functions $f_1(U)$, $f_2(U)$, and $f_3(U)$ are the elements of the matrix $[B(\omega')]$. Thus,

$$\left[Z(\omega')\right]\begin{bmatrix}\frac{\Delta V}{V}\\ \Delta\alpha\\ \Delta\theta\\ \frac{\Delta h}{\bar{c}}\end{bmatrix} = \left[B(\omega')\right] \tag{8}$$

where

$$\left[Z(\omega')\right] = \left[z_{jk}\right] \qquad (j = 1, 2, 3, 4;\ \ k = 1, 2, 3, 4)$$

$$z_{11} = C_{D_V} - C_{T_V}\cos\alpha + 2\mu i\omega'$$

$$z_{12} = C_{D_\alpha} - C_L$$

$$z_{13} = \overline{C}_L$$

$$z_{14} = \bar{c}\beta\left(C_{T_\rho} + C_T\right)\cos\alpha - \bar{c}\,\frac{C_T}{\Delta h}\,i\omega'$$

$$z_{21} = C_{T_V}\sin\alpha + 2\overline{C}_L$$

$$z_{22} = C_T\cos\alpha + C_{L_\alpha} + \left(C_{L_{\alpha'}} + 2\mu\right)i\omega'$$

$$z_{23} = \left(C_{L_{\theta'}} - 2\mu\right)i\omega' + C_{L_{\delta_e}}\frac{\delta_e}{\Delta\theta}\,i\omega'$$

$$z_{24} = -\bar{c}\beta\left(C_{T_\rho} + C_T\right)\sin\alpha$$

$$z_{31} = C_{m_V}$$

$$z_{32} = C_{m_\alpha} + C_{m_{\alpha'}} i\omega'$$

$$z_{33} = 2\mu\left(\frac{k_Y}{\bar{c}}\,\omega'\right)^2 + C_{m_{\theta'}} i\omega' + C_{m_{\delta_e}} \frac{\delta_e}{\Delta\theta}\, i\omega'$$

$$z_{34} = 0$$

$$z_{41} = 0$$

$$z_{42} = 1$$

$$z_{43} = -1$$

and

$$z_{44} = i\omega'$$

The complex matrix $\left[B(\omega')\right]$ is, for unit vertical gust components,

$$\left[B(\omega')\right] = \begin{bmatrix} -\left(C_{D_\alpha} - C_L\right) \\ -\left(C_{L_\alpha} + C_T \cos\alpha + i\omega' C_{L_{\alpha'}}\right) \\ -\left(-C_{m_\alpha} - i\omega' C_{m_{\alpha'}}\right) \\ 0 \end{bmatrix} \tag{9a}$$

and is, for unit horizontal gust components,

$$\left[B(\omega')\right] = \begin{bmatrix} C_{D_V} - C_{T_V} \cos\alpha \\ C_{T_V} \sin\alpha + 2\bar{C}_L \\ -C_{m_V} \\ 0 \end{bmatrix} \tag{9b}$$

The elements of the matrix $\left[B(\omega')\right]$ relate the changes in lift, drag, and pitching moment to changes in angle of attack (see eq. (9a)) and in airspeed (see eq. (9b)) caused by vertical and by horizontal gusts, respectively Vertical gusts cause a change in rms angle of attack given by σ_w/V. The change in rms airspeed caused by horizontal gusts is σ_u/V.

The required frequency-response functions are the elements of the matrix $\left[H(\omega')\right]$ in

$$\left[H(\omega')\right] = \left[Z(\omega')\right]^{-1}\left[B(\omega')\right] \tag{10}$$

Equation (10) is the solution of equation (8) for the variables $\Delta V/V$, $\Delta\alpha$, $\Delta\theta$, and $\Delta h/\bar{c}$.

Substitution of equation (10) and either equation (5) or equation (6) into equation (7) yields the power spectral densities for each of the output variables $\Delta V/V$, $\Delta\alpha$, $\Delta\theta$, and $\Delta h/\bar{c}$. A combination of the frequency-response functions for $\Delta\alpha$ and $\Delta\theta$ gives the frequency-response function for airplane normal-load factor; and $i\omega'$ times the frequency-response function for $\Delta\theta$ gives the frequency-response function for the pitch rate.

Another result from random-process theory is

$$\sigma^2 = \int_0^\infty \Phi(\omega')d\omega' \tag{11}$$

which states that the mean-square value σ^2 of a random variable is the integral of its power spectral density.

Equations (5), (6), (8), and (11) are programed on a digital computer to obtain the rms values of the longitudinal-response variables of an airplane. These variables are σ_V, σ_α, σ_θ, σ_h, σ_q, and σ_n. Obviously, the integration in equation (11) can be carried out only to a finite value of ω'. The upper limit for the integral is large enough to span the frequencies of all the airplane modes and large enough so that further increase does not appreciably increase the value of the integral.

RESULTS OF CALCULATIONS

Airplane data on mass and dimensions are listed in table I. The airplane is a medium jet transport about the size of a Boeing 720B. Pertinent aerodynamic data are listed in table II for altitudes of 6.096 km and 10.668 km and a Mach number range of 0.6 to 0.85. Combinations of feedback gains used in the present study are as follows:

Pitch autopilot		Autothrottle	Combined pitch autopilot and autothrottle		
K_θ	K_q	K_t	K_θ	K_q	K_t
0.59	0.8	0.0026	0.59	0.8	0.0260
1.18	1.6	.0260			
2.36	3.2	.1300			
4.72	6.4				

Stability Calculations

The stability of each airplane-autopilot combination is determined by examination of the sign of the real parts of the roots of the characteristic equation of each system. It is necessary to assure that all systems are stable because the analysis leading to equation (7) is correct only if the system is stable. The airplane with the automatic controls not operating has an unstable phugoid for all flight conditions. This instability is caused by elimination of the major phugoid damping term C_D from the equations of motion. (See the appendix.) A low-gain pitch-angle feedback $\left(K_\theta = 0.236\right)$ is used to stabilize the phugoid. Most other combinations of airplane, automatic controls, and flight conditions are stable. At an altitude of 6.096 km and a Mach number of 0.85, one mode associated with the pitch autopilot $\left(K_\theta = 4.72 \text{ and } K_q = 6.4\right)$ is unstable.

Responses to Turbulence

Responses of the airplane-autopilot combinations to both unit rms horizontal and vertical components of turbulence $\left(\text{that is, } \sigma_w = \sigma_u = 1 \text{ m/sec}\right)$ are calculated for combinations of feedback gains and flight conditions. The responses to unit rms vertical gust components are presented for altitudes of 6.096 km and 10.668 km in figures 4 to 7; responses to unit horizontal components, in figures 8 to 11. Although the rms variations of the variables are plotted as functions of Mach number, it should be emphasized that the calculations are made at discrete Mach numbers with the assumption that the airplane is in equilibrium at those Mach numbers. The plots permit comparisons of the responses at the various Mach numbers (calculations are made at $M = 0.6$, 0.7, 0.8, and 0.85) and permit interpolation to obtain responses at intermediate Mach numbers.

DISCUSSION

The responses of the airplane to unit rms components of gust velocity $\left(\sigma_w = \sigma_u = 1 \text{ m/sec}\right)$ are useful for analysis of the automatic-control operation. Because the equations are linear (see eqs. (5) to (7)), responses to any degree of turbulence may be obtained by use of appropriate multiples of unit σ_w and σ_u (that is, of 1 m/sec).

Responses to Vertical Gusts

Basic airplane.- The responses of the basic stabilized airplane ($K_\theta = 0.236$ and $K_q = 0$) to unit rms vertical gust components are presented in figure 4. The rms variations of airspeed σ_V, angle of attack σ_α, attitude angle σ_θ, and pitch rate σ_q are relatively small for all flight conditions. However, the rms variations of altitude σ_h and normal-load factor σ_n are considered to be excessively large This qualitative description of the responses of the basic airplane is sufficient for analysis of the performance of the automatic control system. The purpose of the analysis is to determine which feedback quantities can be used effectively to reduce excursions of the motion variables from a desired trim condition without increasing structural loads on the airplane.

Effect of pitch autopilot.- The responses of the airplane with the pitch autopilot operating (design gains $K_\theta = 2.36$ and $K_q = 3.2$) are also shown in figure 4. Since the purpose of the pitch autopilot is to maintain the attitude angle at some desired value, it is not surprising that the rms variation in attitude angle σ_θ is reduced as shown. This autopilot also produced sizable reductions of the rms variation of angle of attack σ_α, pitch rate σ_q, and normal-load factor σ_n. The effect of the pitch autopilot on rms variations of airspeed σ_V and altitude σ_h depends on Mach number and altitude. For any flight condition, the rms variation of altitude σ_h is not appreciably smaller than for the basic airplane; and, for most flight conditions, operation of the pitch autopilot results in an increased rms variation of airspeed σ_V.

Doubling the design gains of the pitch autopilot (that is, $K_\theta = 4.72$ and $K_q = 6.4$) does not produce an appreciable improvement in the response of the airplane to unit rms vertical gust components. (See fig. 5.) Proportionate decreases in the rms variations of attitude angle σ_θ and pitch rate σ_q are obtained but the other variables, except airspeed, are only slightly increased or decreased. The rms variation of airspeed σ_V generally is proportionately increased. A proportionate increase or decrease of the rms value of a variable means that either doubling or halving the gains affects the value of the variable by approximately the same amount.

When $K_\theta = 4.72$, $K_q = 6.4$, and $h = 6.096$ km, the rms variations of attitude angle σ_θ and pitch rate σ_q have the unusual trend with Mach number shown by the curves labeled (4) in figure 5(a). In the section entitled "Stability Calculations," it is stated that a control mode is unstable at $M = 0.85$ and at $h = 6.096$ km. The damping of this mode decreases as the trim Mach number is increased. By way of explanation, the lightly damped mode can result in an increase of the power-spectral density at the natural frequency of the mode which, in turn, produces larger rms values of the variables. It should be recalled that the mean-square value of a variable is the integral of its power-spectral density. (See eq. (11).)

In practice, it is recommended that autopilot gains be reduced during flight in turbulence. Use of a pitch autopilot with larger than design gains cannot be recommended on the basis of the data presented herein.

The rms variations of angle of attack σ_{α}, attitude angle σ_{θ}, pitch rate σ_q, and normal-load factor σ_n increase when the pitch autopilot gains are reduced from the design values of $K_{\theta} = 4.72$ and $K_q = 6.4$. However, the values of these rms variables, with $K_{\theta} = 0.59$ and $K_q = 0.8$, are smaller than the corresponding values for the basic airplane. (Compare curves labeled (1) of figs. 4 and 5.) Reducing the gains produces only small changes in the rms variation of altitude σ_h; but the rms variation of airspeed σ_V is reduced to approximately the values for the basic airplane. The pitch autopilot with gains $K_{\theta} = 0.59$ and $K_q = 0.8$ is considered to provide improved responses.

Effect of autothrottle.- Responses to unit rms vertical gust components for the basic airplane and for the airplane with the autothrottle in operation are compared in figure 6. The rms variations of all the variables except airspeed σ_V and altitude σ_h are practically unchanged by operation of the autothrottle. Increasing the autothrottle gain K_t provides a large reduction in rms variation of altitude σ_h. The thrust changes required to hold altitude, however, cause an increase of the rms variation of airspeed σ_V. The autothrottle gain $K_t = 0.0260$ gives a sizable reduction of the rms variation of altitude σ_h without a large increase of the rms variation of airspeed σ_V. So, $K_t = 0.0260$ is adjudged an improvement.

Effect of pitch autopilot combined with autothrottle.- The responses of the airplane to vertical gusts with the combined pitch autopilot and autothrottle in operation $\left(K_{\theta} = 0.59, K_q = 0.8, \text{ and } K_t = 0.0260\right)$ are compared with the responses of the basic airplane in figure 7. The rms variations of angle of attack σ_{α}, attitude angle σ_{θ}, pitch rate σ_q, and normal-load factor σ_n with the combined controls in operation are essentially the same as for the operation of the pitch autopilot alone $\left(K_{\theta} = 0.59 \text{ and } K_q = 0.8\right)$. (See fig. 5.) The rms variations of airspeed σ_V and altitude σ_h with the combined controls in operation are essentially the same as for the operation of the autothrottle alone $\left(K_t = 0.0260\right)$. (See fig. 6.) However, the rms variation of airspeed σ_V at an altitude of 6.096 km is much larger with the combined controls in operation than with the autothrottle alone in operation.

Responses to Horizontal Gusts

The responses of the airplane to unit rms horizontal gust components ($\sigma_u = 1$ m/sec) are characterized by relatively small rms variations of the variables σ_{α}, σ_{θ}, σ_h, σ_q, and σ_n. (See figs. 8 to 11.) The rms variation of airspeed σ_V is as large or larger than for the response to unit rms vertical gust components. The rms variation of altitude σ_h is less than 10 meters for all flight conditions but, in severe turbulence, this variation

may be more than desired. In figures 8 to 11, the rms variations of pitch rate σ_q are not accurately represented. The calculated values of σ_q are so small that, without a change of scale, the curves lie almost entirely on the axis at $\sigma_q = 0$. Therefore, the lines shown in figures 8 to 11 represent the largest calculated value of σ_q. The rms variations of angle of attack σ_α, attitude angle σ_θ, and pitch rate σ_q are almost negligible even in severest turbulence. The effect of the automatic control system on the responses to unit rms horizontal gust components is similar to the effect of the automatic control system on the responses to unit rms vertical gust components.

Assessment of Control Systems

In the responses of the airplane to unit rms vertical gust components, the automatic control system with selected feedback gains produces the desired reduction in excursions of altitude without increasing the normal-load factor. However, excursions in airspeed are increased. The pitch autopilot is not effective in reducing excursions in altitude; and the autothrottle, although quite effective in reducing excursions in altitude, causes increased excursions in airspeed.

The relevance of the control effectiveness is shown by consideration of a practical situation. It is assumed that the airplane is trimmed for $M = 0.85$ at $h = 10.668$ km and is flying in severe turbulence (that is, $\sigma_W = 9$ m/sec). The proximity to the buffet boundary for the basic airplane is illustrated in figure 12(a) by the parallelogram surrounding the trim condition. The rms variation of altitude is $\sigma_h = 297$ m and the rms variation of airspeed is $\sigma_V = 1.7$ knots The curve labeled $n = 1\ 53$ is the buffet boundary for a normal-load factor of 1.53. (This corresponds to the calculated rms variation of normal-load factor $\sigma_n = 0.53$.) With the pitch autopilot in operation $\left(K_\theta = 0.59\right.$ and $\left.K_q = 0.8\right)$, the airplane response to severe turbulence is illustrated by the parallelogram with solid lines in figure 12(b). The rms variation in altitude is increased to $\sigma_h = 333$ m and the rms variation of airspeed is increased to $\sigma_V = 1.8$ knots. However, the rms normal-load factor is reduced to $\sigma_n = 0.46$. The effect of adding the autothrottle $\left(K_t = 0.0260\right)$ is illustrated by the parallelogram with dashed lines in figure 12(b). The rms variation of normal-load factor remains the same; but the rms variation of altitude is reduced to $\sigma_h = 144$ m or about one-half that of the basic airplane and the rms airspeed is further increased to $\sigma_V = 3.4$ knots.

In this severe turbulence ($\sigma_W = 9$ m/sec), σ_V, σ_h, and σ_n remain below the corresponding buffet boundary for the airplane, even with the automatic controls operating. Extreme gust velocities, however, can be three times their rms values Thus, with altitude excursions between about 430 m and 1000 m, depending on the control system, the buffet boundary is crossed. Also, the limit normal-load factor of 2.5 is exceeded.

The responses of the airplane shown in figure 12 for $\sigma_W = 9$ m/sec may be considered to represent the extreme condition, for which the gust component is three times its rms value of 3 m/sec. The value of $\sigma_W = 3$ m/sec characterizes mild turbulence. It is not likely that the buffet boundary is crossed while the airplane is flying in this mild turbulence; but the altitude excursions, without altitude hold, are sufficient to cause the airplane to exceed its allowable altitude corridor of 600 m. Thus, the danger of collision is increased. Further, the wide variations of normal-load factor can be uncomfortable for passengers.

CONCLUDING REMARKS

The present study has shown that, with proper selection of feedback gains, an automatic control system can reduce excursions in altitude of a jet transport flying in turbulence without increasing structural loads. The control system uses feedback of attitude-angle and pitch-rate signals to the elevator to provide attitude hold and uses feedback of altitude and altitude-rate signals to the throttle to provide altitude hold. Without the altitude-hold control, altitude excursions of the airplane are sufficient, in mild turbulence, to cause the airplane to exceed its allowable altitude corridor. The altitude-hold mode is effective in reducing altitude excursions by about 50 percent. In extremely severe turbulence the airplane exceeds its limit normal-load factor with the control system in operation.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., July 26, 1972.

APPENDIX

EQUATIONS FOR AIRPLANE LONGITUDINAL MOTION

It is assumed that the mass, a constant, of an airplane is concentrated at the origin of a system of moving wind axes as shown in figure 1. Changes in the moment of inertia about the Y-axis of the airplane $I_Y = mk_Y{}^2$ are assumed to be negligible. Aerodynamic and applied force components act along the X- and Z-axes with a moment about the Y-axis. The airplane is free to move in the X- and Z-directions and to rotate about the Y-axis. The equations of longitudinal motion, starting from equilibrium, are obtained by expanding, in a Taylor's series, the following equations describing Newton's laws of motion:

$$f_X(V,\dot{V},\alpha,\theta,h,U,\delta_t) = F_X - m\dot{V} = 0$$

$$f_Z(V,\dot{V},\alpha,\theta,h,\dot{\alpha},\dot{\theta},\delta_e,U,\delta_t) = F_Z + mV(\dot{\theta} - \dot{\alpha}) = 0$$

$$M_Y(V,\dot{V},\alpha,\dot{\alpha},\dot{\theta},U,\delta_e) = M_Y - mk_Y{}^2\ddot{\theta} = 0$$

Dependence of the variables on time is implicit in these equations. In the series expansion, the first terms and the coefficients of the remaining terms are evaluated at equilibrium conditions. Also, forces and moments caused by control movement and by atmospheric turbulence are placed on the right-hand side of the equations. An equation for rate of climb is given since force variations with changes in altitude are included. For small disturbances, only the first two terms of the series need be retained. The resulting linear equations are given as follows:

$$\left.\begin{aligned}
&F_X - m\dot{V} + V\frac{\partial F_X}{\partial V}\frac{\Delta V}{V} + \frac{\partial F_X}{\partial \alpha}\Delta\alpha + \frac{\partial F_X}{\partial \theta}\Delta\theta + \bar{c}\frac{\partial F_X}{\partial h}\frac{\Delta h}{\bar{c}} - mV\frac{\Delta\dot{V}}{V} = -F_X(\delta_t,U) \\
&F_Z + mV(\dot{\theta} - \dot{\alpha}) + V\frac{\partial F_Z}{\partial V}\frac{\Delta V}{V} + \frac{\partial F_Z}{\partial \alpha}\Delta\alpha + \frac{\partial F_Z}{\partial \dot{\alpha}}\Delta\dot{\alpha} + \frac{\partial F_Z}{\partial \theta}\Delta\theta + \frac{\partial F_Z}{\partial \dot{\theta}}\Delta\dot{\theta} + \bar{c}\frac{\partial F_Z}{\partial h}\frac{\Delta h}{\bar{c}} \\
&\quad + mV(\dot{\theta} - \dot{\alpha})\frac{\Delta V}{V} + mV(\Delta\dot{\theta} - \Delta\dot{\alpha}) = -F_Z(\delta_e,U) \\
&M_Y - mk_Y{}^2\ddot{\theta} + V\frac{\partial M_Y}{\partial V}\frac{\Delta V}{V} + \frac{\partial M_Y}{\partial \alpha}\Delta\alpha + \frac{\partial M_Y}{\partial \dot{\alpha}}\Delta\dot{\alpha} + \frac{\partial M_Y}{\partial \dot{\theta}}\Delta\dot{\theta} - mk_Y{}^2\Delta\ddot{\theta} = -M_Y(\delta_e,U) \\
&\dot{h} - V\sin(\theta - \alpha) - V\sin(\theta - \alpha)\frac{\Delta V}{V} + V\cos(\theta - \alpha)\Delta\alpha - V\cos(\theta - \alpha)\Delta\theta + \bar{c}\frac{\Delta\dot{h}}{\bar{c}} = 0
\end{aligned}\right\} \quad (A1)$$

Equilibrium conditions are characterized by zero acceleration and zero rate of climb (that is, $\dot{V} = V\dot{\theta} = V\dot{\alpha} = \ddot{\theta} = \dot{h} = 0$). This condition leads to

$$F_X = 0$$

$$F_Z = 0$$

$$M_Y = 0$$

$$\theta = \alpha$$

Also, it is assumed that $F_X(\delta_t,U) = F_Z(\delta_e,U) = M_Y(\delta_e,U) = 0$ at equilibrium.

Explicitly, the forces and moments are given by the following equations:

$$\left.\begin{aligned} F_X &= \frac{\rho}{2} SV^2\left[C_T(V,h)\cos\alpha - C_D(\alpha,V) - \overline{C}_L \sin(\theta - \alpha)\right] \\ F_Z &= \frac{\rho}{2} SV^2\left[-C_T(V,h)\sin\alpha - C_L(\alpha,\dot{\alpha},\dot{\theta}) + \overline{C}_L \cos(\theta - \alpha)\right] \\ M_Y &= \frac{\rho}{2} S\bar{c}V^2\, C_m(V,\alpha,\dot{\alpha},\dot{\theta}) \end{aligned}\right\} \tag{A2}$$

and the equilibrium conditions are as follows:

$$\left.\begin{aligned} C_D &= C_T \cos\alpha \\ C_L &= \overline{C}_L - C_T \sin\alpha \\ C_m &= 0 \end{aligned}\right\} \tag{A3}$$

The desired linear differential equations are obtained by use of equations (A3) and by substitution of equations (A2) into equations (A1). The desired forms of the equations include the following:

(1) After substitution, force equations are divided by $\frac{\rho}{2} SV^2$ and moment equations, by $\frac{\rho}{2} S\bar{c}V^2$

(2) Time t is changed to nondimensional time $t' = \frac{tV}{\bar{c}}$ with derivatives with respect to t' denoted by a prime and thus, for instance, $\frac{d\alpha}{dt} = \frac{V}{\bar{c}} \frac{dV}{d\frac{tV}{\bar{c}}} = \frac{V}{\bar{c}} \alpha'$

(3) Mass is nondimensionalized as $\mu = \frac{m}{\rho S\bar{c}}$

(4) Change of air density with altitude is $\beta = -\frac{1}{\rho}\frac{d\rho}{dh}$ which comes from $\rho = \rho_o e^{-\beta h}$ when ρ_o is air density at standard sea-level conditions

The resulting equations are as follows:

$$\left.\begin{aligned}
&2\mu\frac{\Delta V'}{V} - \left(C_{T_V}\cos\alpha - C_{D_V}\right)\frac{\Delta V}{V} + \left(C_{D_\alpha} - C_L\right)\Delta\alpha + \overline{C}_L\,\Delta\theta + \beta\bar{c}\Big(C_{T_\rho} \\
&\quad + C_T\Big)\cos\alpha\,\frac{\Delta h}{\bar{c}} = -C_{T_{\delta_t}}\delta_t + f_1(U) \\
&\left(C_{T_V}\sin\alpha + 2\overline{C}_L\right)\frac{\Delta V}{V} + \left(C_{L_\alpha} + C_T\cos\alpha\right)\Delta\alpha + \left(2\mu + C_{L_{\alpha'}}\right)\Delta\alpha' + \left(C_{L_{\theta'}} - 2\mu\right)\Delta\theta' \\
&\quad - \bar{c}\beta\left(C_{T_\rho} + C_T\right)\sin\alpha\,\frac{\Delta h}{\bar{c}} = -C_{L_{\delta_e}}\delta_e + f_2(U) \\
&C_{m_V}\frac{\Delta V}{V} + C_{m_\alpha}\,\Delta\alpha + C_{m_{\alpha'}}\,\Delta\alpha' + C_{m_{\theta'}}\,\Delta\theta' - 2\mu\frac{k_Y^2}{\bar{c}^2}\,\Delta\theta'' = -C_{m_{\delta_e}}\delta_e + f_3(U) \\
&\Delta\alpha - \Delta\theta + \frac{\Delta h'}{\bar{c}} = 0
\end{aligned}\right\} \tag{A4}$$

For power-fixed flight, the coefficient of $\Delta V/V$ in the first of equations (A4) is $+\left(C_D + C_{D_V}\right)$. It is this term that provides most of the damping of the phugoid. The C_D-part of the term is eliminated by the thrust component for powered flight and, consequently, the phugoid is likely to be unstable without some means of stability augmentation. The Laplace transforms of equations (A4) are taken with the Laplace variable $\lambda = \frac{\bar{c}}{V}s$ to obtain the following equations:

$$\left.\begin{aligned}
&\left(2\mu\lambda - C_{T_V}\cos\alpha + C_{D_V}\right)\frac{\Delta V}{V} - \left(C_L - C_{D_\alpha}\right)\Delta\alpha + \overline{C}_L\,\Delta\theta + \bar{c}\beta\left(C_{T_\rho} + C_T\right)\cos\alpha\,\frac{\Delta h}{\bar{c}} = -C_{T_{\delta_t}}\delta_t + f_1(U) \\
&\left(C_{T_V}\sin\alpha + 2\overline{C}_L\right)\frac{\Delta V}{V} + \left[C_T\cos\alpha + C_{L_\alpha} + \left(C_{L_{\alpha'}} + 2\mu\right)\lambda\right]\Delta\alpha + \left(C_{L_{\theta'}} - 2\mu\right)\lambda\,\Delta\theta \\
&\quad - \bar{c}\beta\left(C_{T_\rho} + C_T\right)\sin\alpha\,\frac{\Delta h}{\bar{c}} = -C_{L_{\delta_e}}\delta_e + f_2(U) \\
&C_{m_V}\frac{\Delta V}{V} + \left(C_{m_\alpha} + C_{m_{\alpha'}}\lambda\right)\Delta\alpha + \left(C_{m_{\theta'}}\lambda - 2\mu\frac{k_Y^2}{\bar{c}^2}\lambda^2\right)\Delta\theta = -C_{m_{\delta_e}}\delta_e + f_3(U) \\
&\Delta\alpha - \Delta\theta + \lambda\frac{\Delta h}{\bar{c}} = 0
\end{aligned}\right\} \tag{A5}$$

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TABLE I.- AIRPLANE DATA

Mass, kg	79 379
Wing area, m^2	236.9
Mean aerodynamic chord, m	5.86
Wing span, m	40.69
Moment of inertia about Y-axis, kg-m^2	4.74

TABLE II.- AIRPLANE AERODYNAMIC DATA

	Altitude = 6 096 m				Altitude = 10 668 m			
	M = 6	M = 7	M = .8	M = 85	M = 6	M = 7	M = 8	M = 85
α, deg	1.57550	.78530	.19000	.01200	4.02540	2.39510	1.20810	.83710
C_L	.27837	.20452	.15658	.13870	.54258	.39863	.30520	.27035
C_D	.01770	.01520	.01570	.01635	.02970	.02170	.01870	.01895
C_T	.01771	.01620	.01570	.01635	.02977	.02172	.01870	.01895
C_{L_α}, l/rad	4.38313	4.36594	4.52637	4.53783	4.77274	4.84722	5.13943	5.19673
$V\partial C_D/\partial V$	0.00000	0.00000	0.00000	.03800	0.00000	0.00000	.01000	.06170
C_{D_α}, l/rad	.06589	.04822	0.00000	0.00000	.13984	.10434	.08470	.07587
$V\partial C_T/\partial V$	.00362	.00571	.00762	.00891	.00609	.00765	.00907	.01032
$V\partial C_m/\partial V$	0.00000	0.00000	.01875	-.01875	0.00000	0.00000	.01875	-.01875
C_{m_α}, l/rad	-.76776	-.69901	-.72766	-.80214	-.96257	-.95111	-1.07143	-1.15737
$C_{m_{\dot\alpha}}$, l/rad	-.74843	-.77063	-.83222	-.87519	-.78925	-.82792	-.91100	-.97546
$C_{m_{\dot\theta}}$, l/rad	-1.54152	-1.61144	-1.61144	-1.61144	-1.78190	-1.79766	-1.84206	-1.88503
$C_{L_{\delta e}}$, l/rad	.21199	.20054	.17762	.17189	.21199	.20054	.17762	.17189
$C_{m_{\delta e}}$, l/rad	-.76203	-.76203	-.76203	-.76203	-.76203	-.76203	-.76203	-.76203

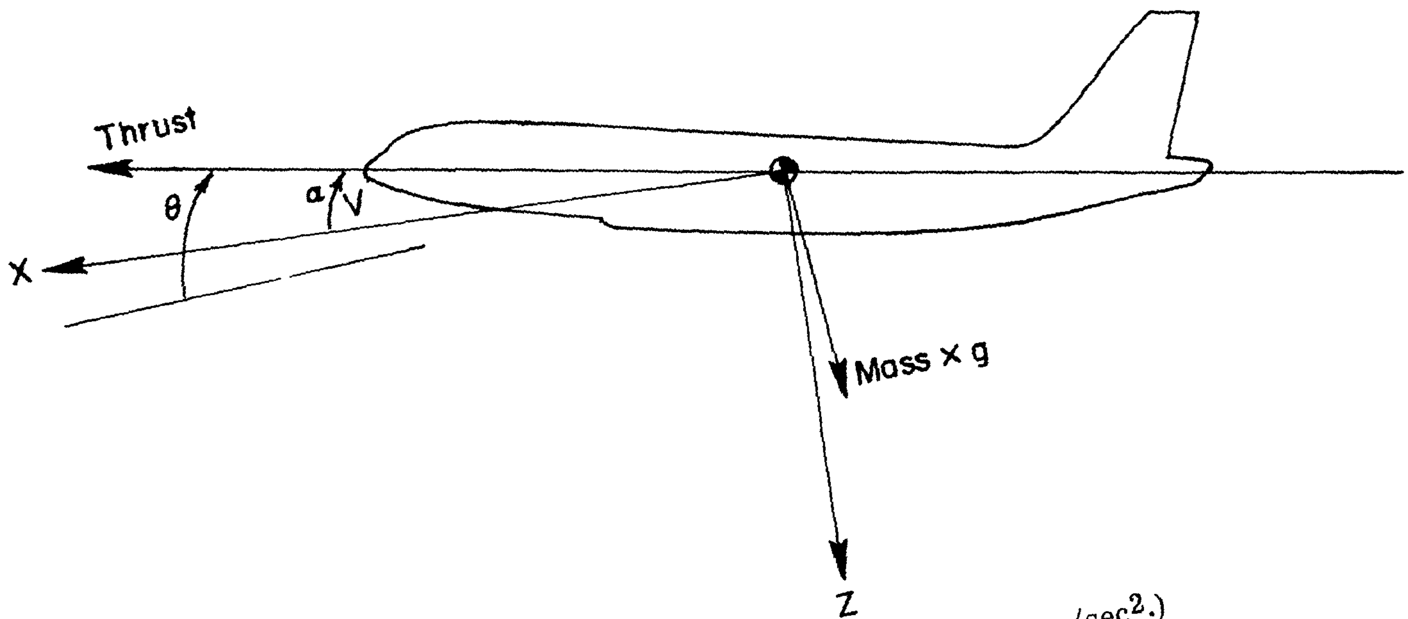


Figure 1.- Wind-axes system. (1g = 9.807 m/sec^2.)

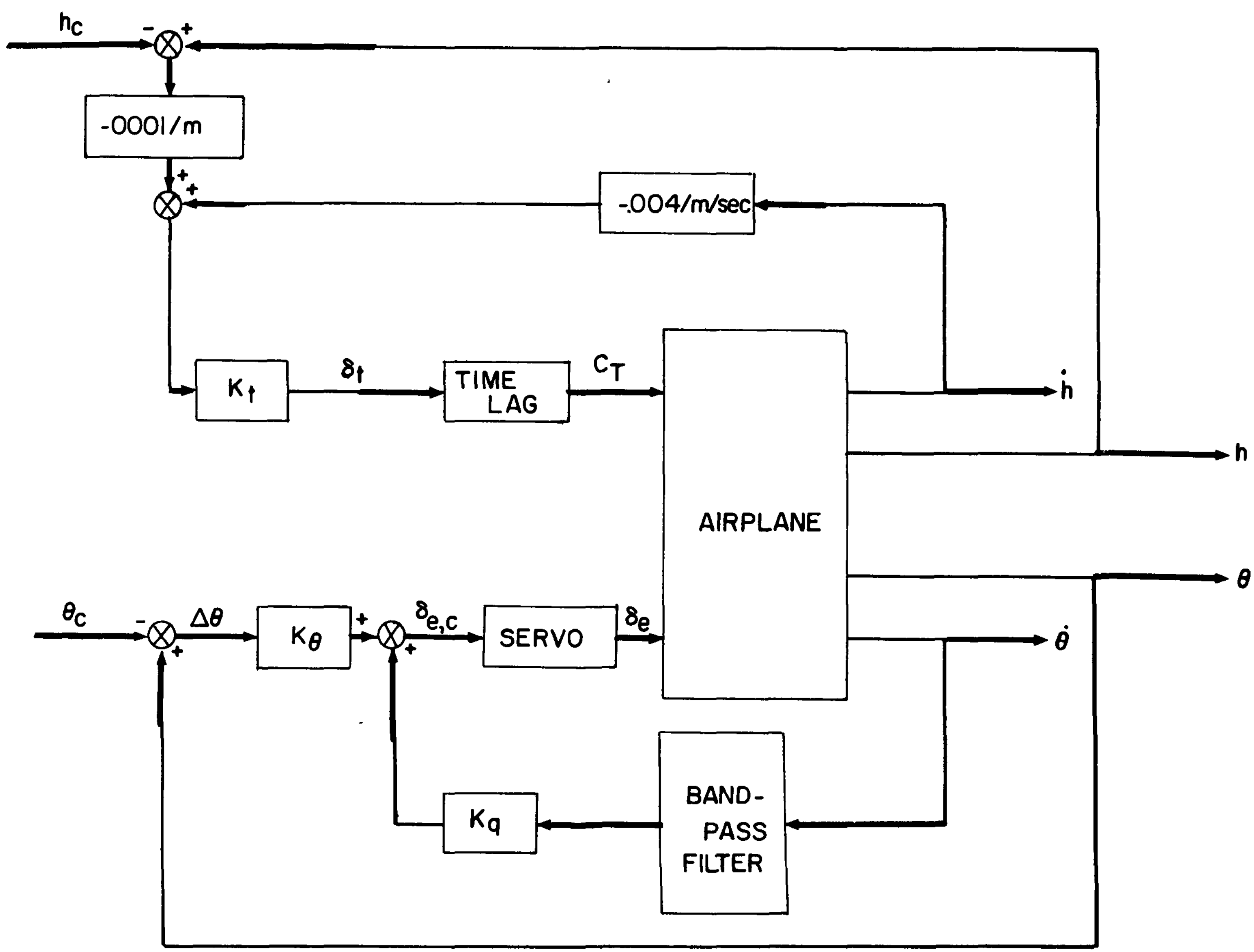


Figure 2.- Block diagram of automatic control system.

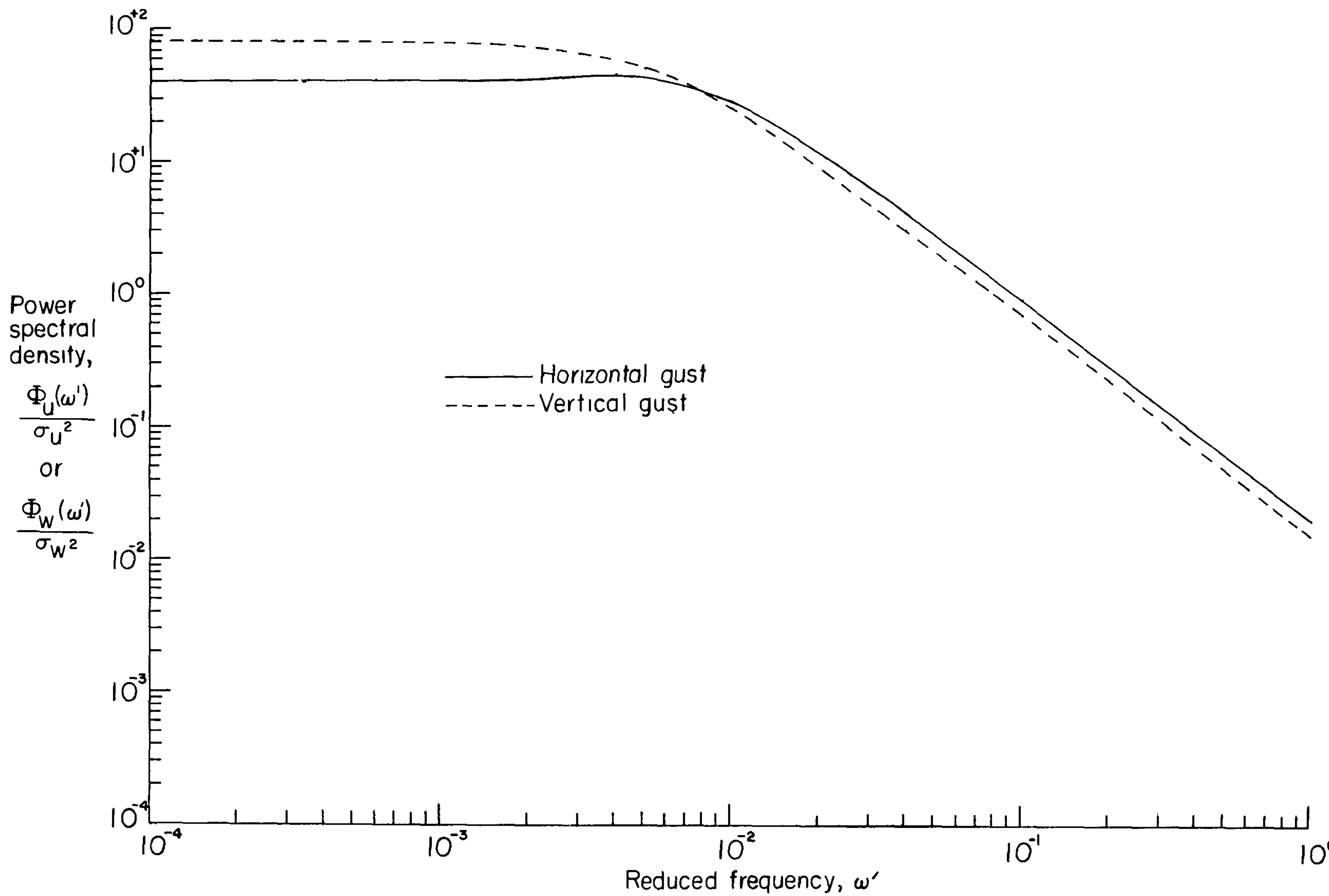
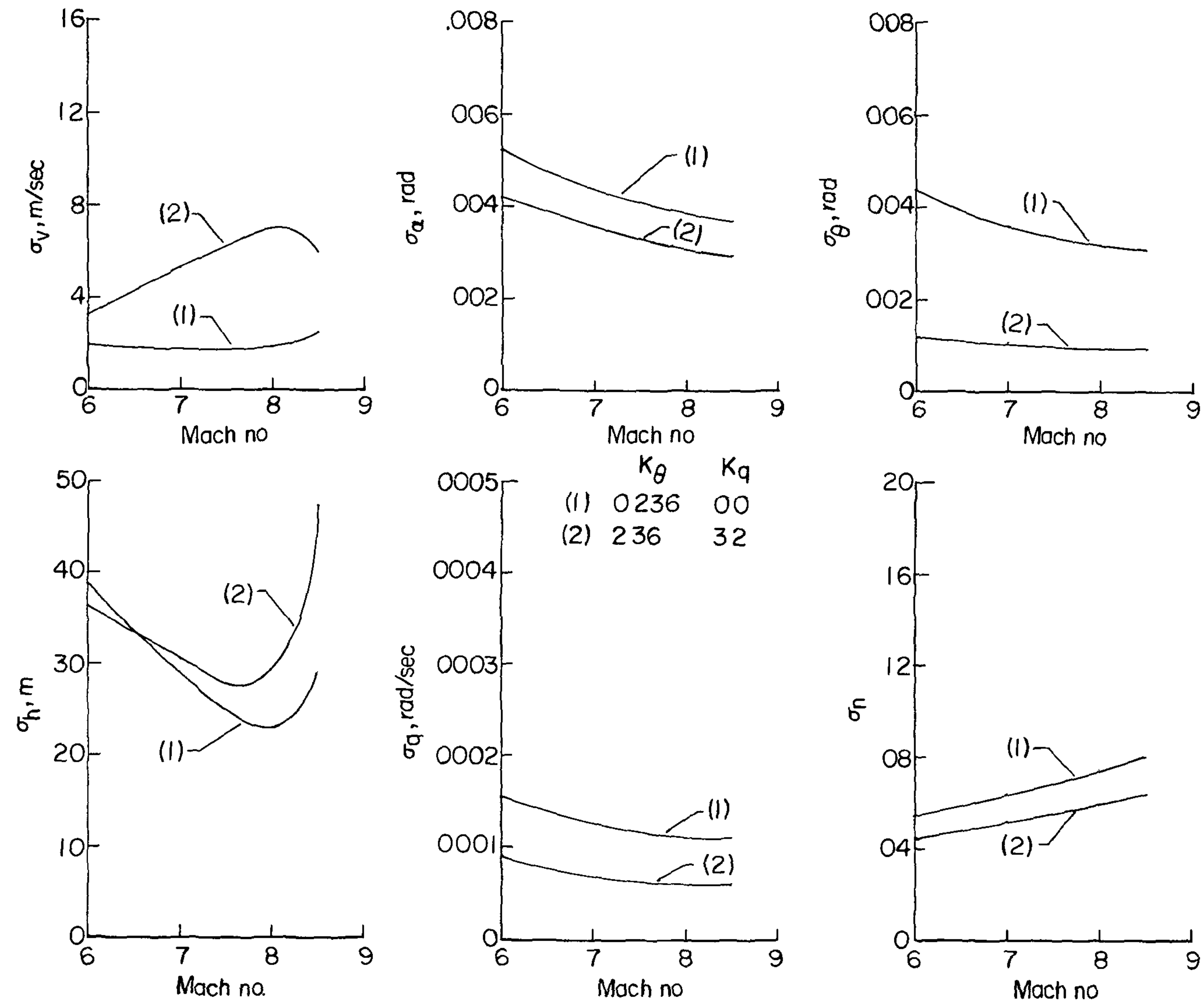
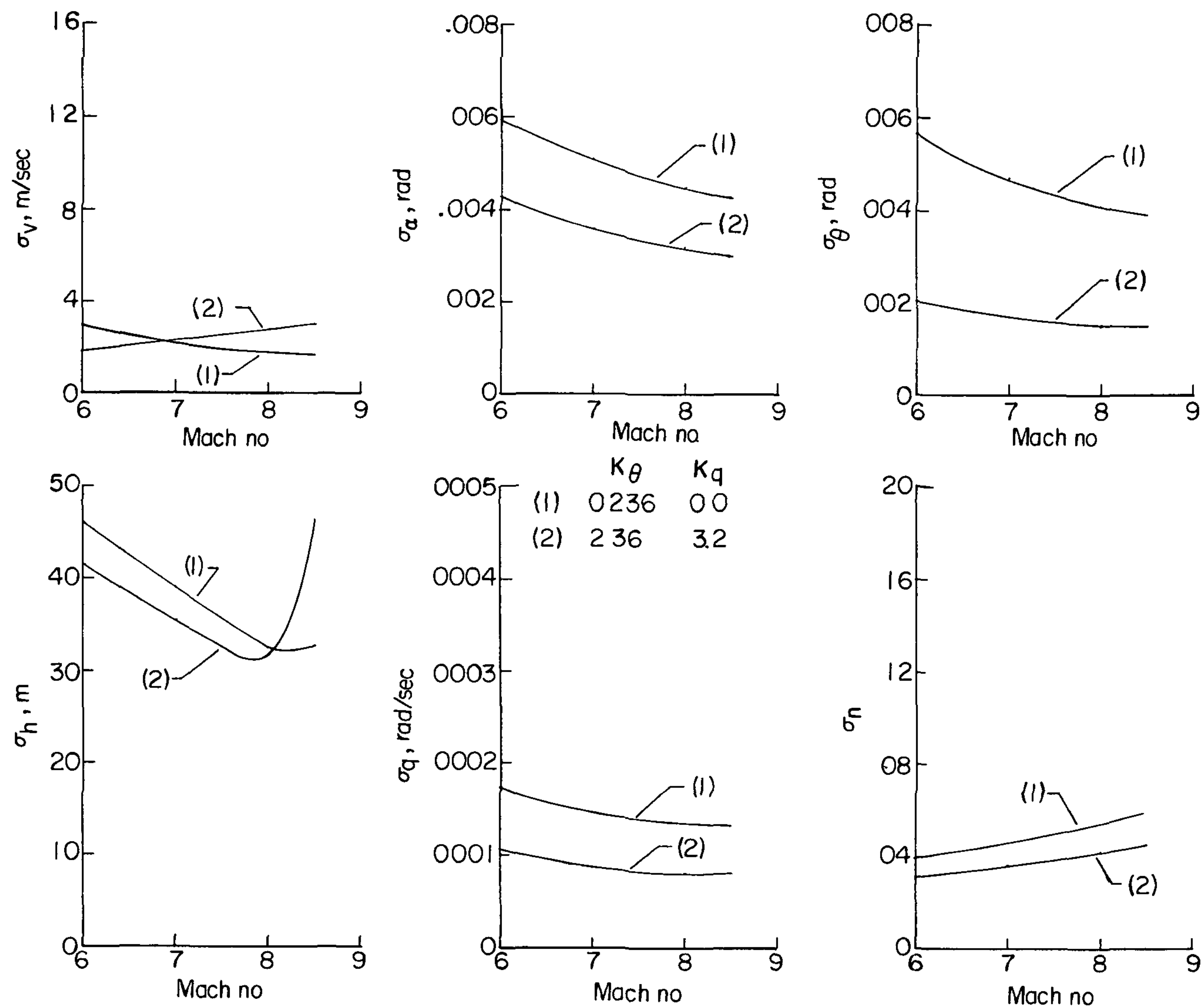


Figure 3.- Analytical representation of atmospheric-turbulence power spectra.



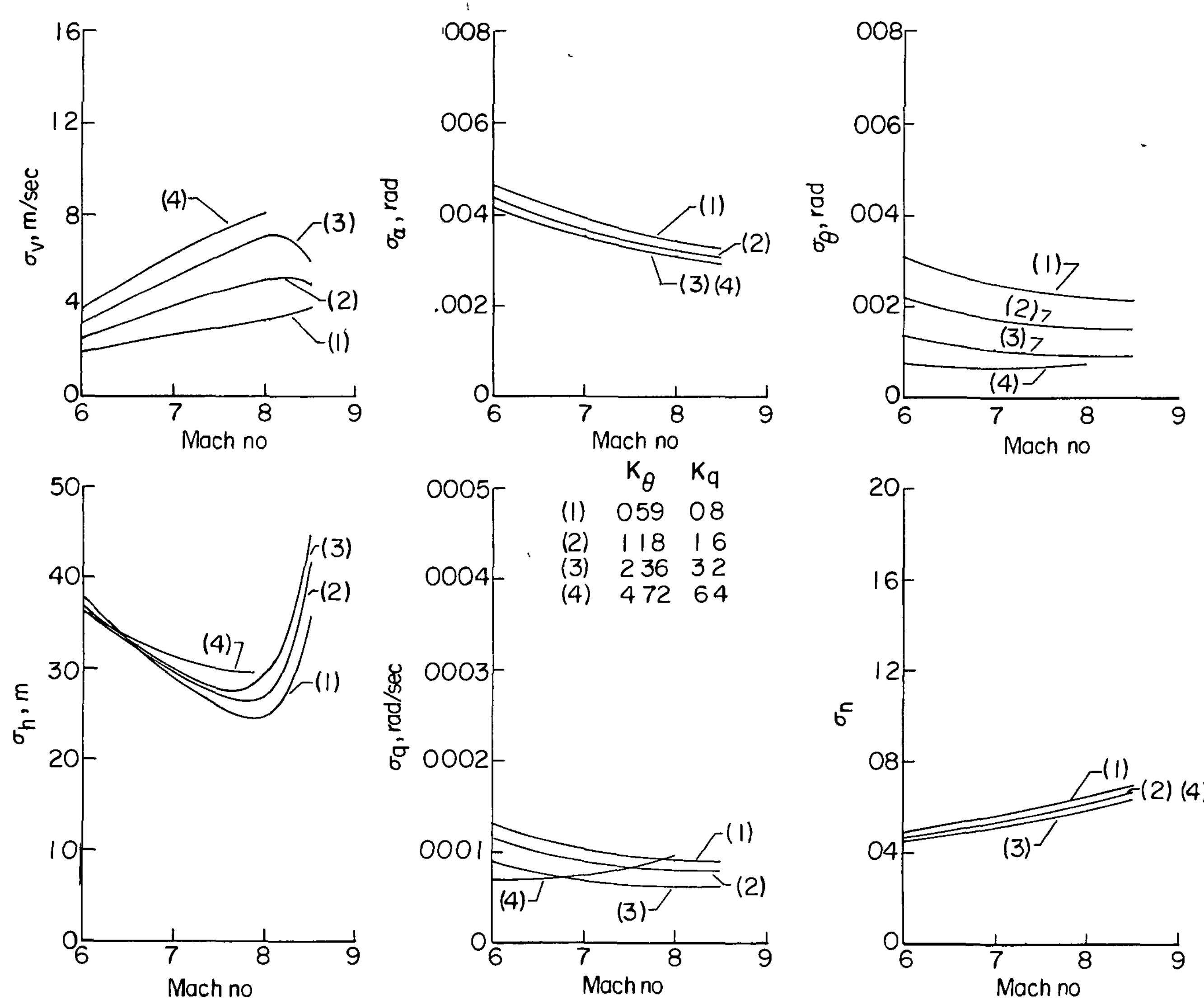
(a) Altitude = 6.096 km.

Figure 4.- Effect of pitch autopilot on responses of airplane to vertical gusts. σ_w = 1 m/sec.



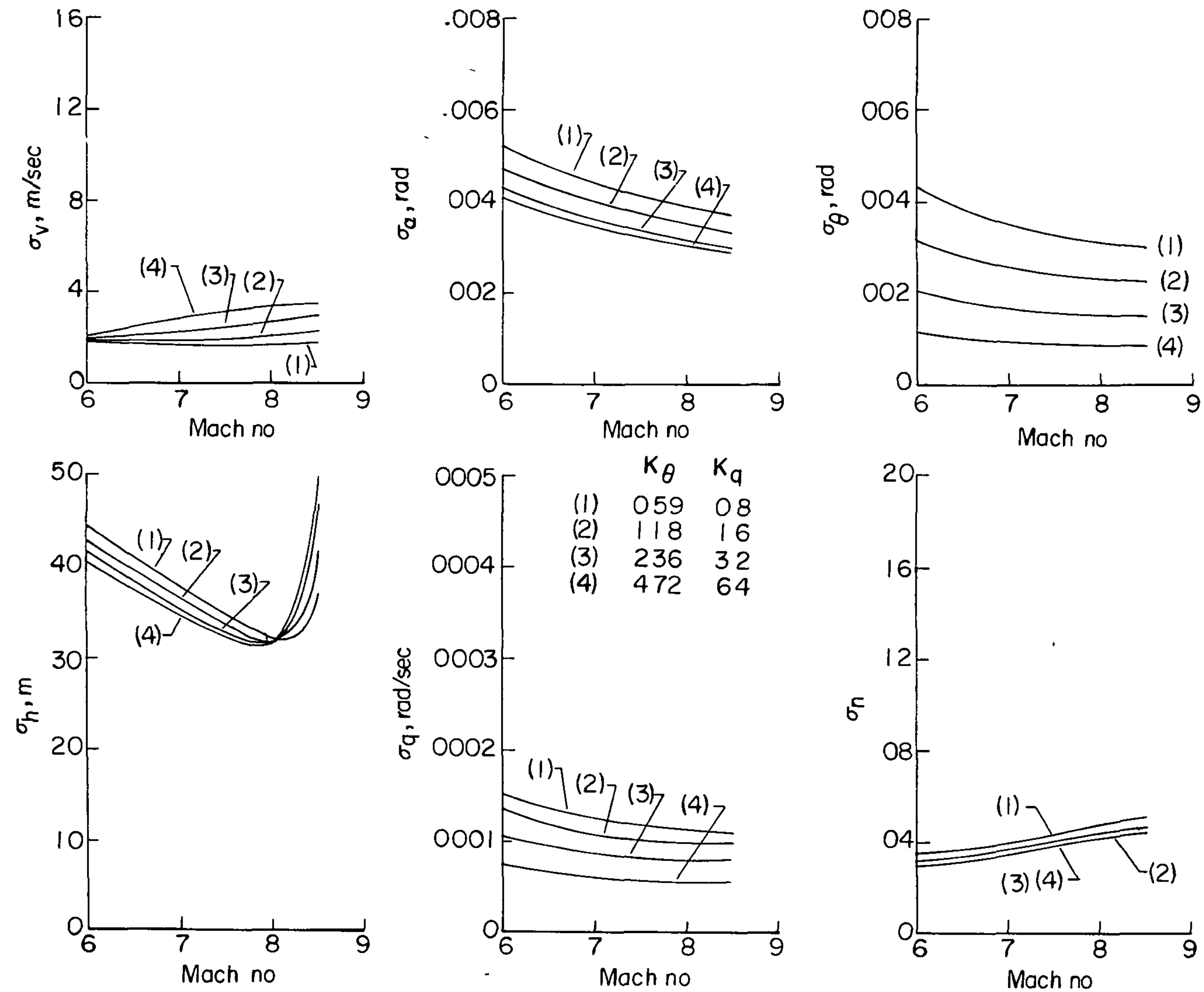
(b) Altitude = 10.668 km.

Figure 4.- Concluded.



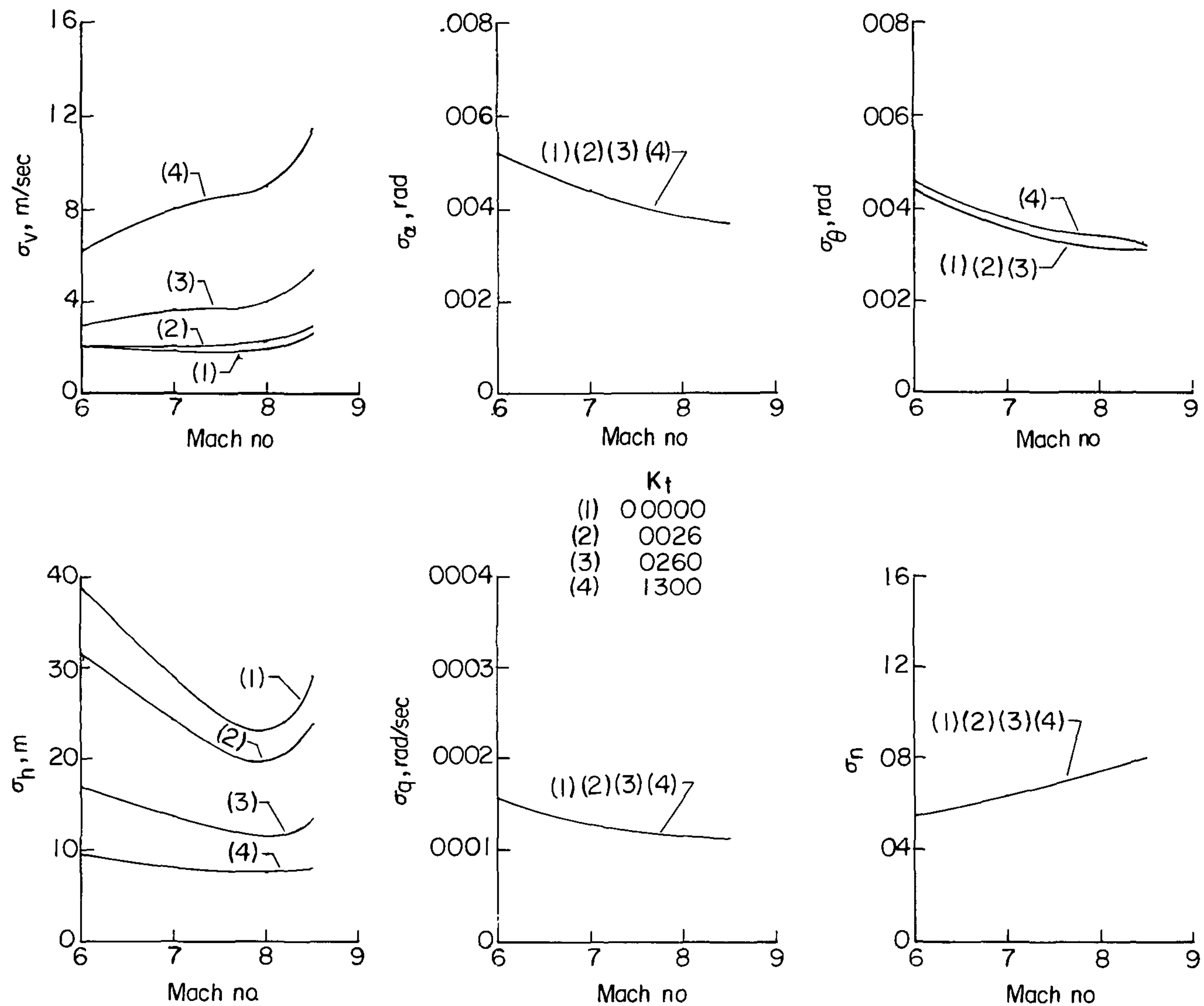
(a) Altitude = 6.096 km.

Figure 5.- Effect of pitch-autopilot gain on responses of airplane to vertical gusts. $\sigma_W = 1$ m/sec.



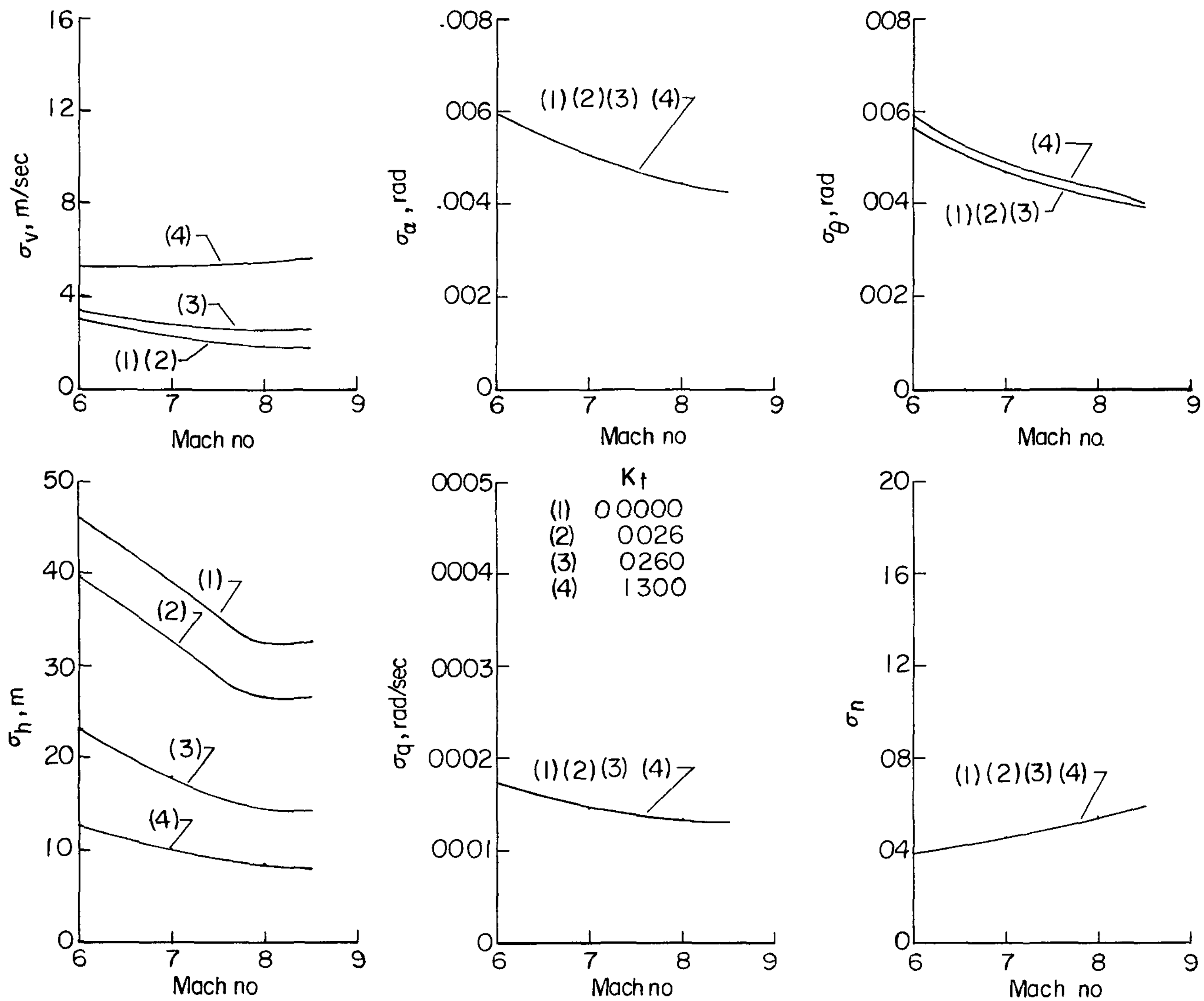
(b) Altitude = 10.668 km.

Figure 5.- Concluded.



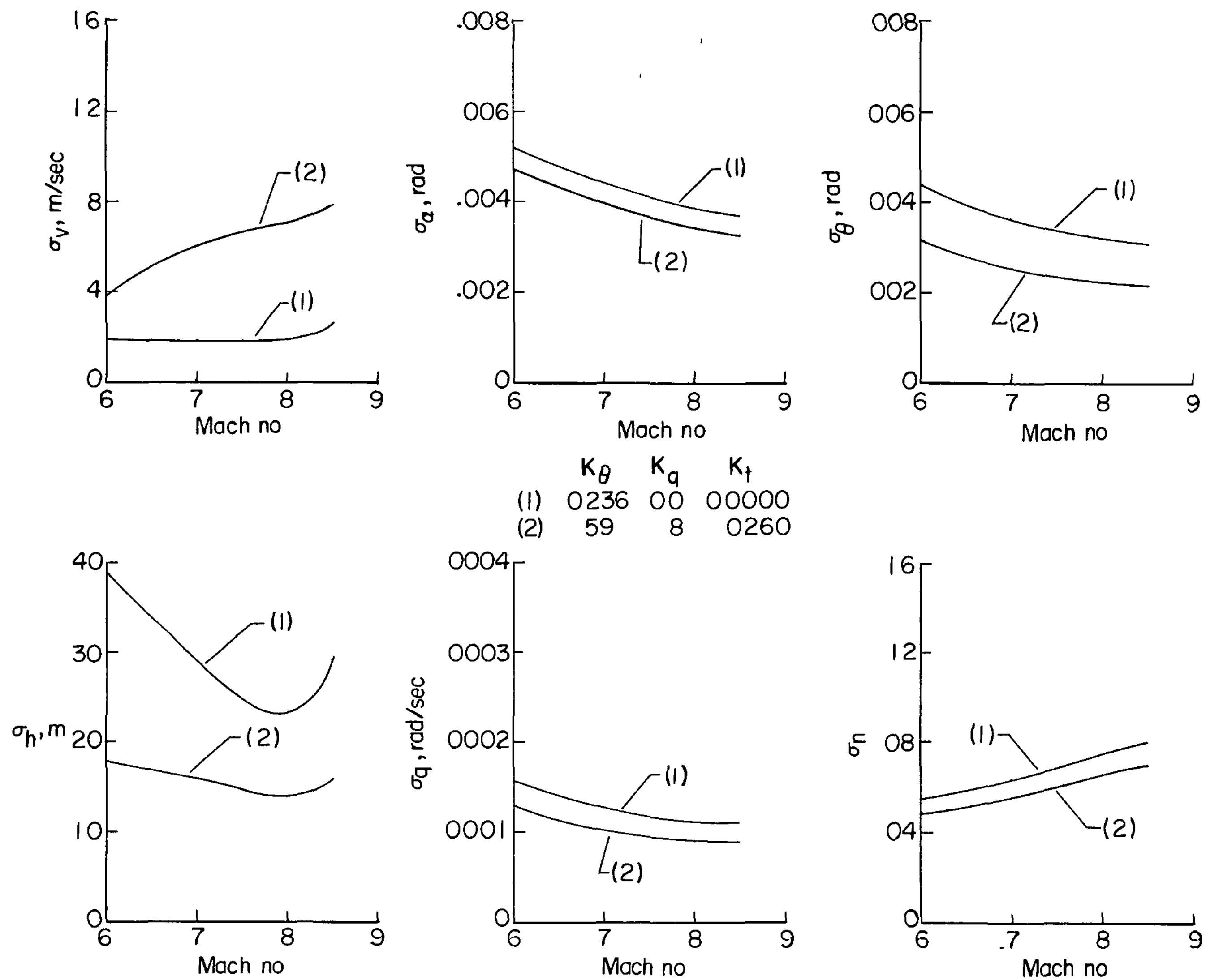
(a) Altitude = 6.096 km.

Figure 6.- Effect of autothrottle on responses of airplane to vertical gusts. $\sigma_W = 1$ m/sec.



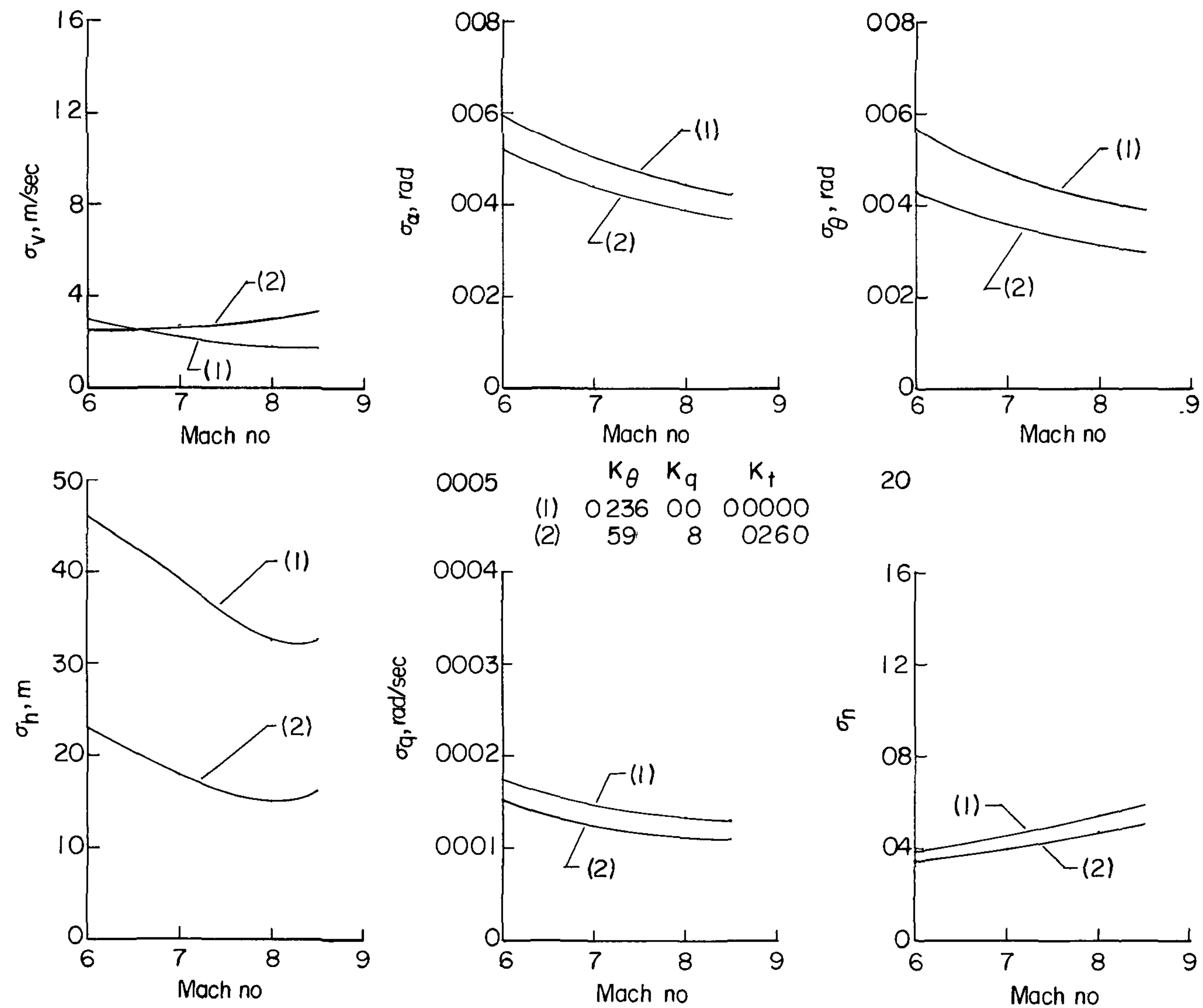
(b) Altitude = 10.668 km.

Figure 6.- Concluded.



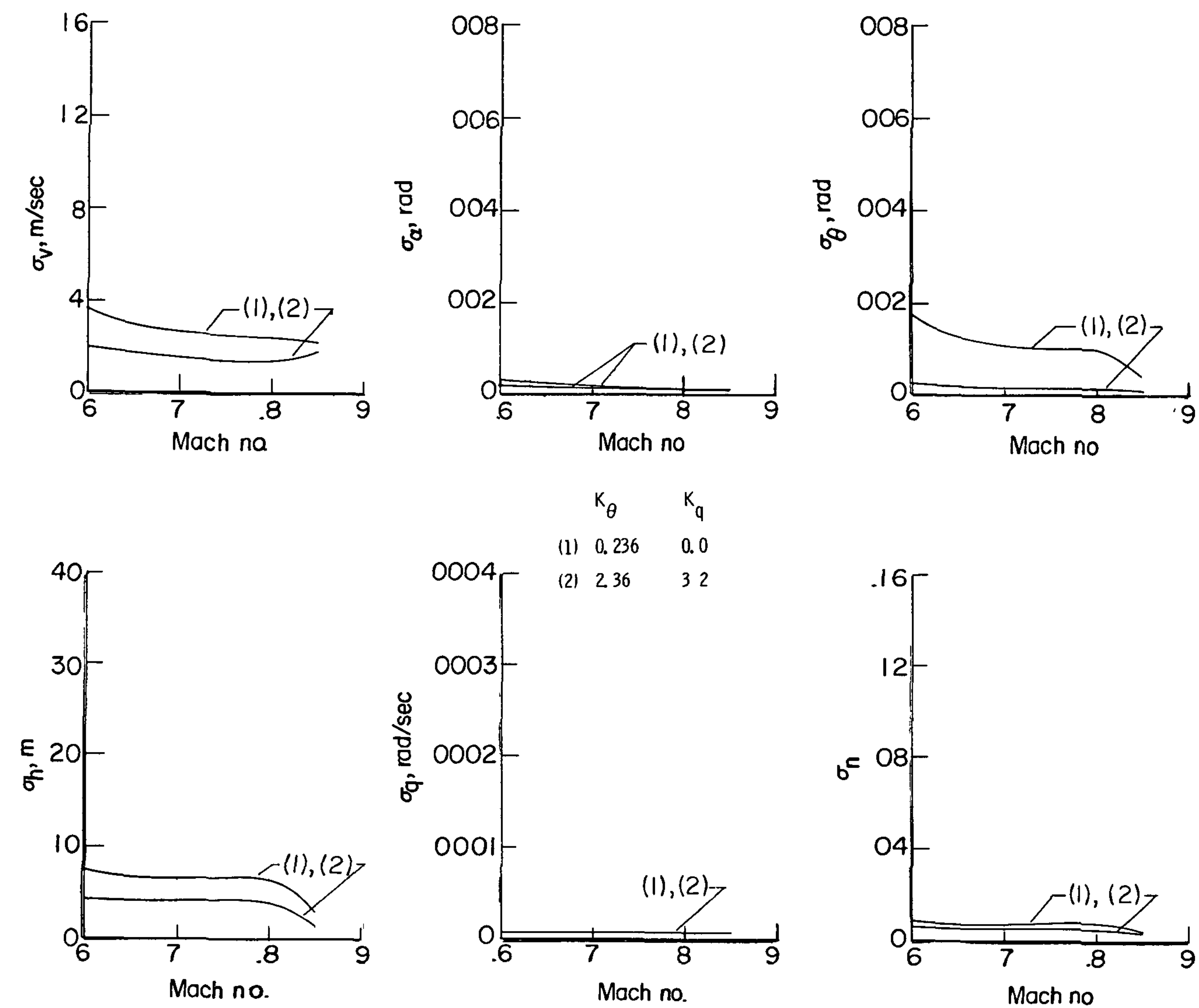
(a) Altitude = 6.096 km.

Figure 7.- Effect of combined pitch autopilot and autothrottle on responses of airplane to vertical gusts. σ_w = 1 m/sec.



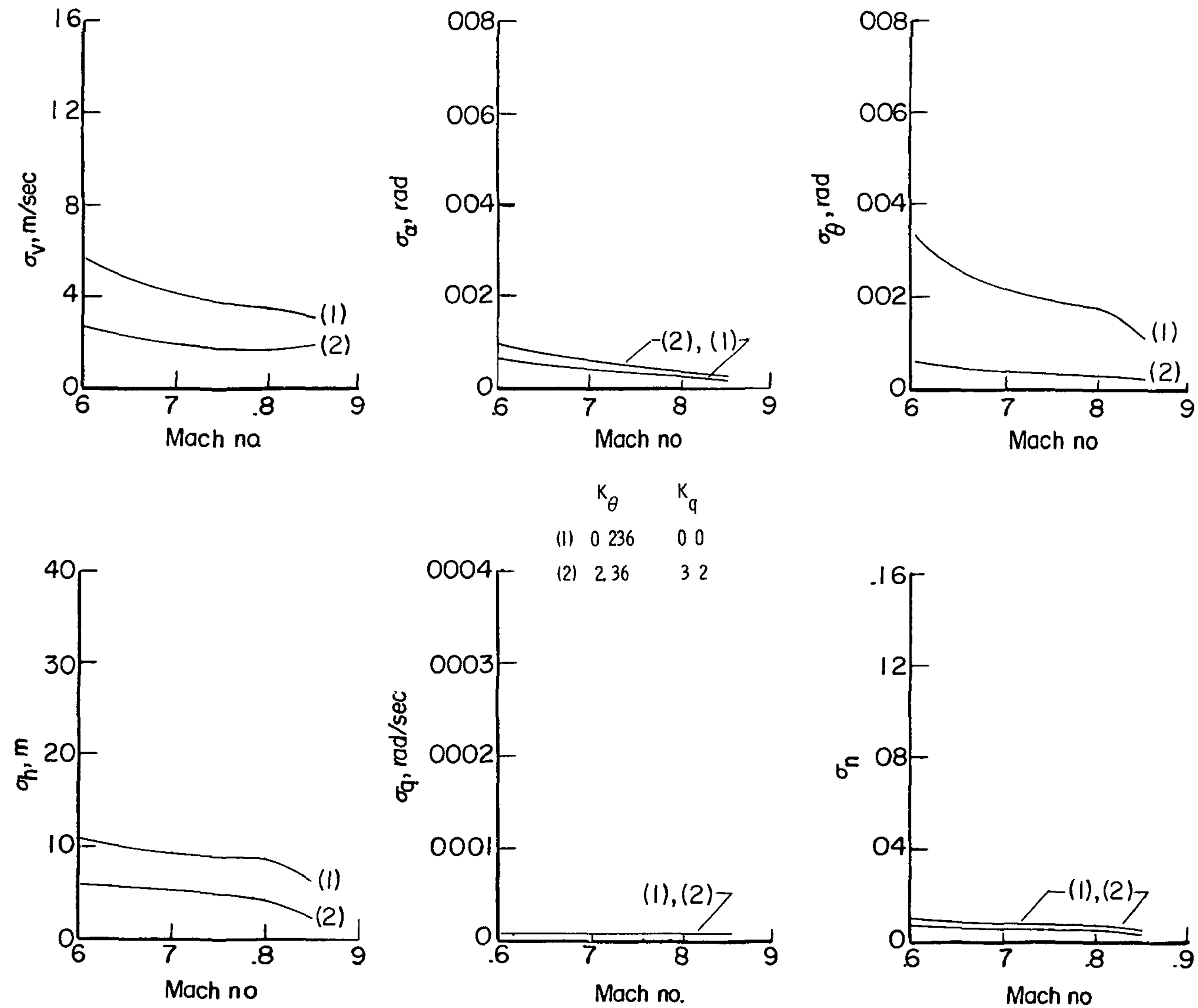
(b) Altitude = 10.668 km.

Figure 7.- Concluded.



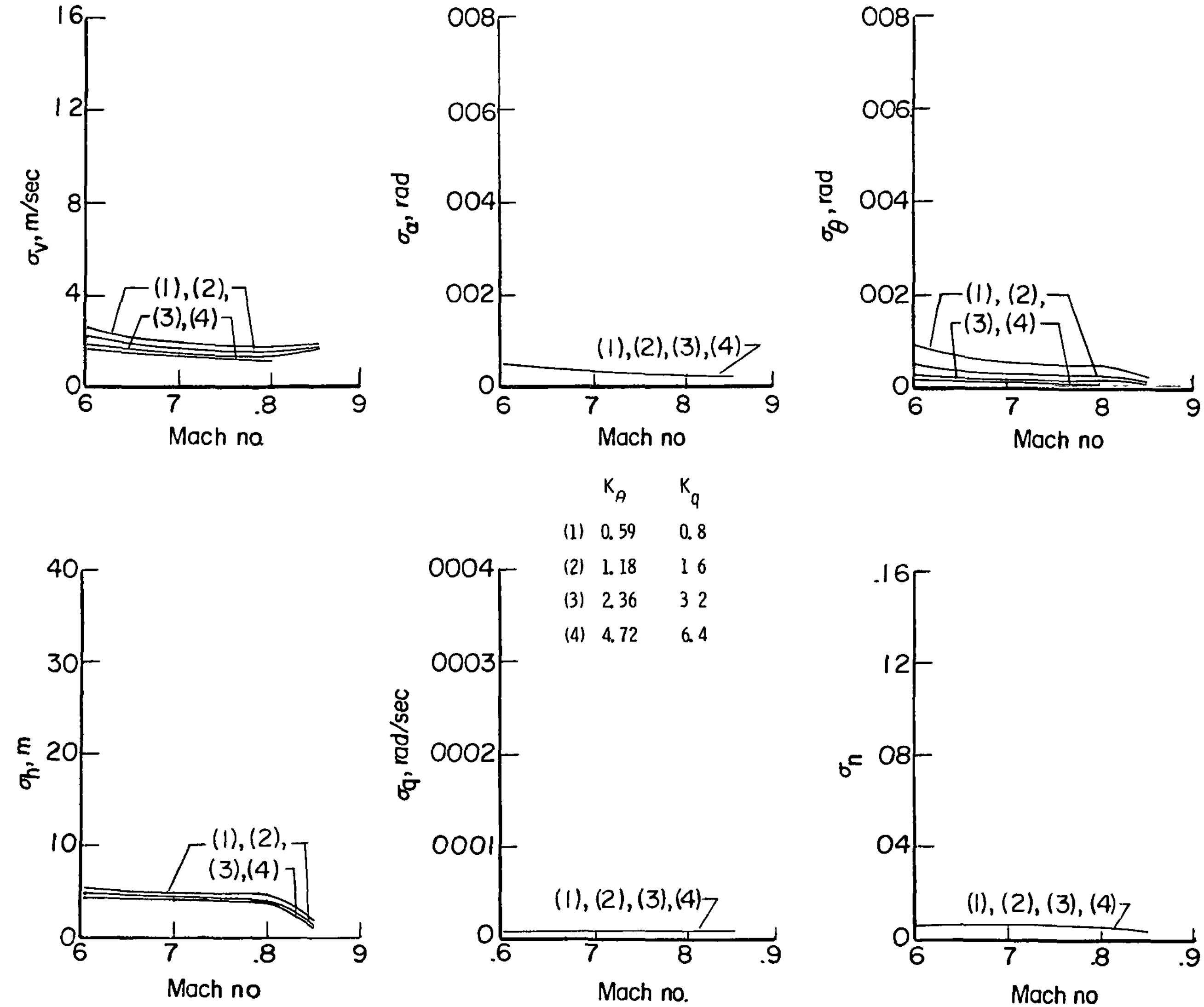
(a) Altitude = 6.096 km.

Figure 8.- Effect of pitch autopilot on responses of airplane to horizontal gusts. σ_u = 1 m/sec.



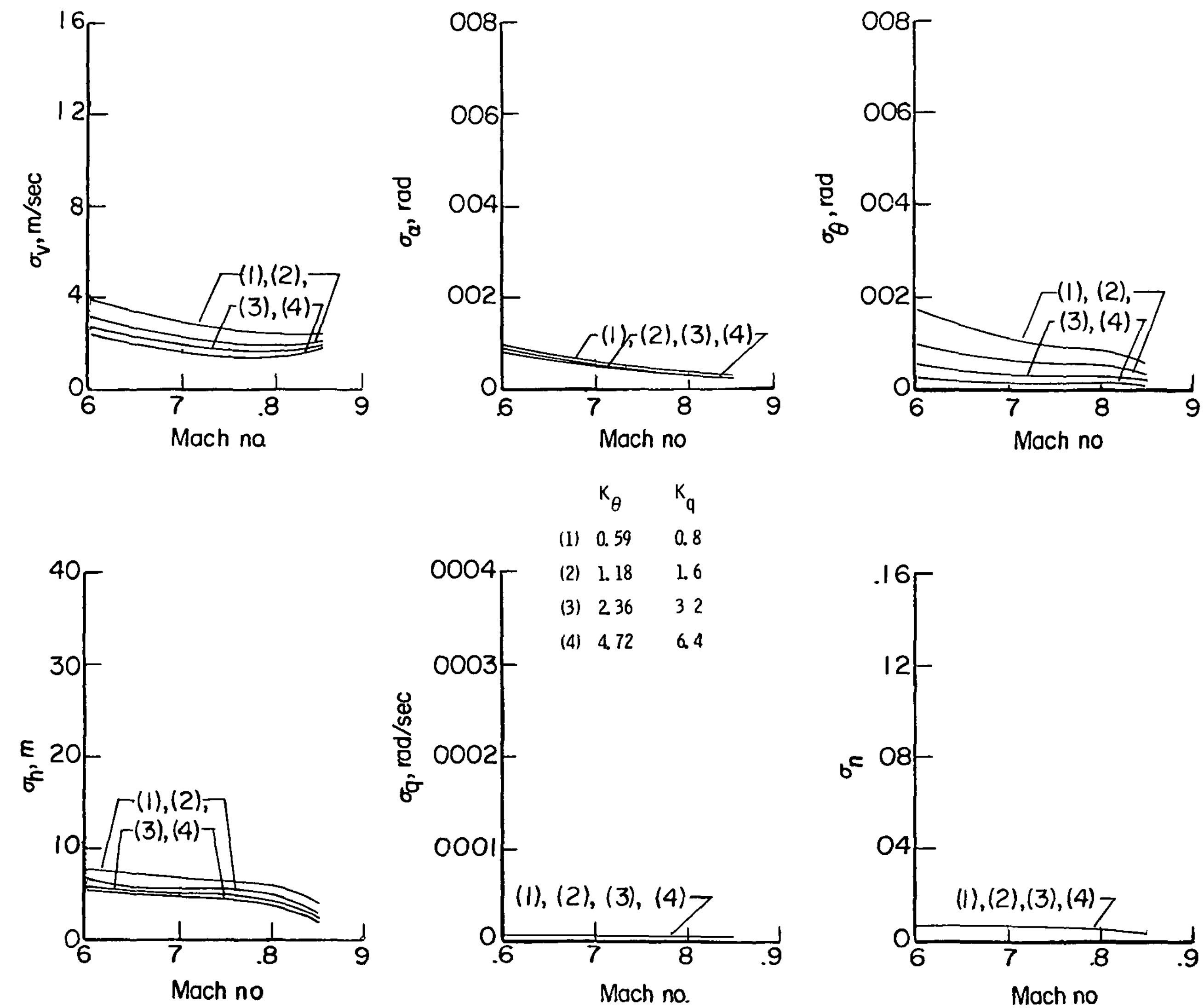
(b) Altitude = 10.668 km.

Figure 8.- Concluded.



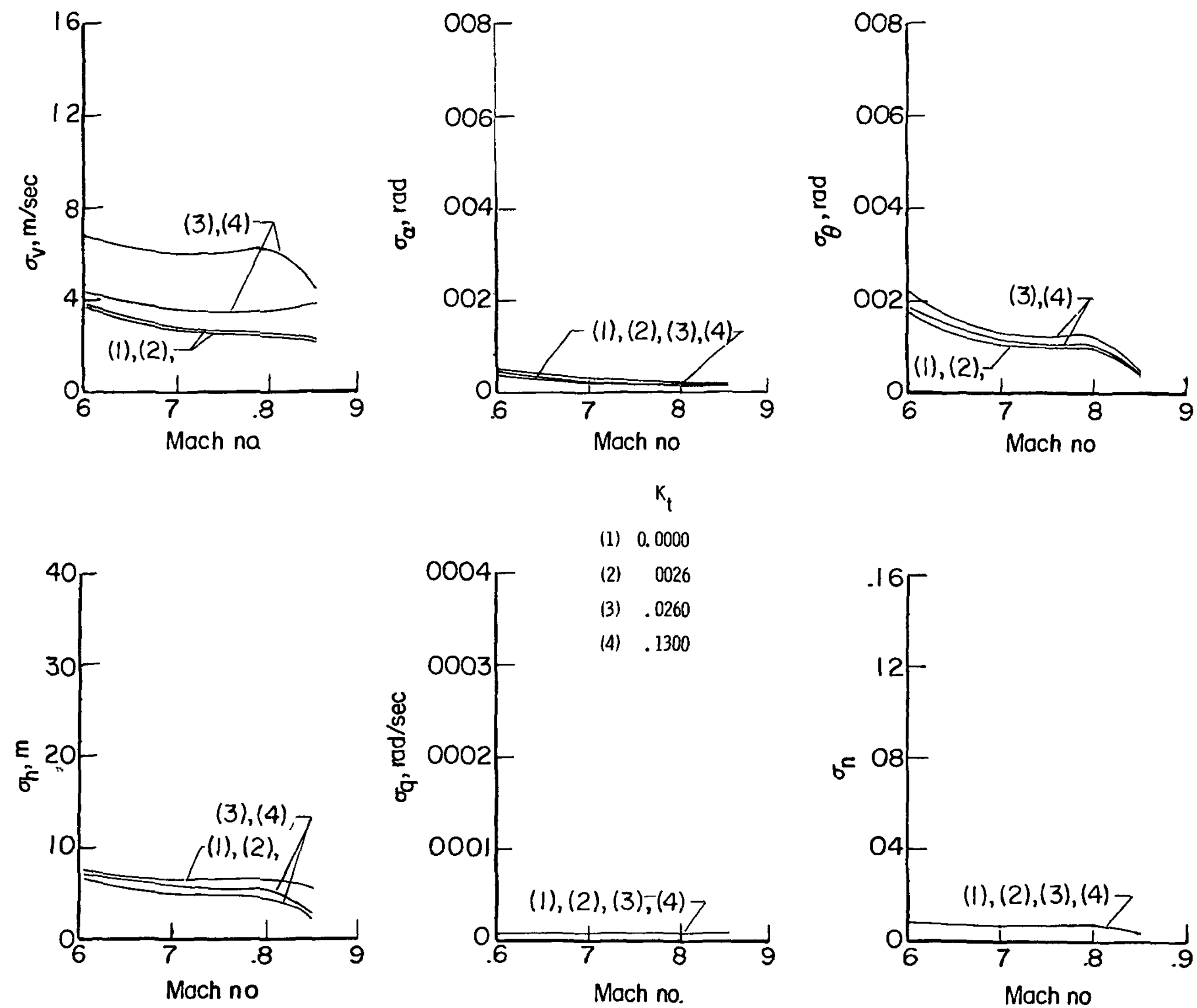
(a) Altitude = 6.096 km.

Figure 9.- Effect of pitch-autopilot gain on responses of airplane to horizontal gusts $\sigma_u = 1$ m/sec.



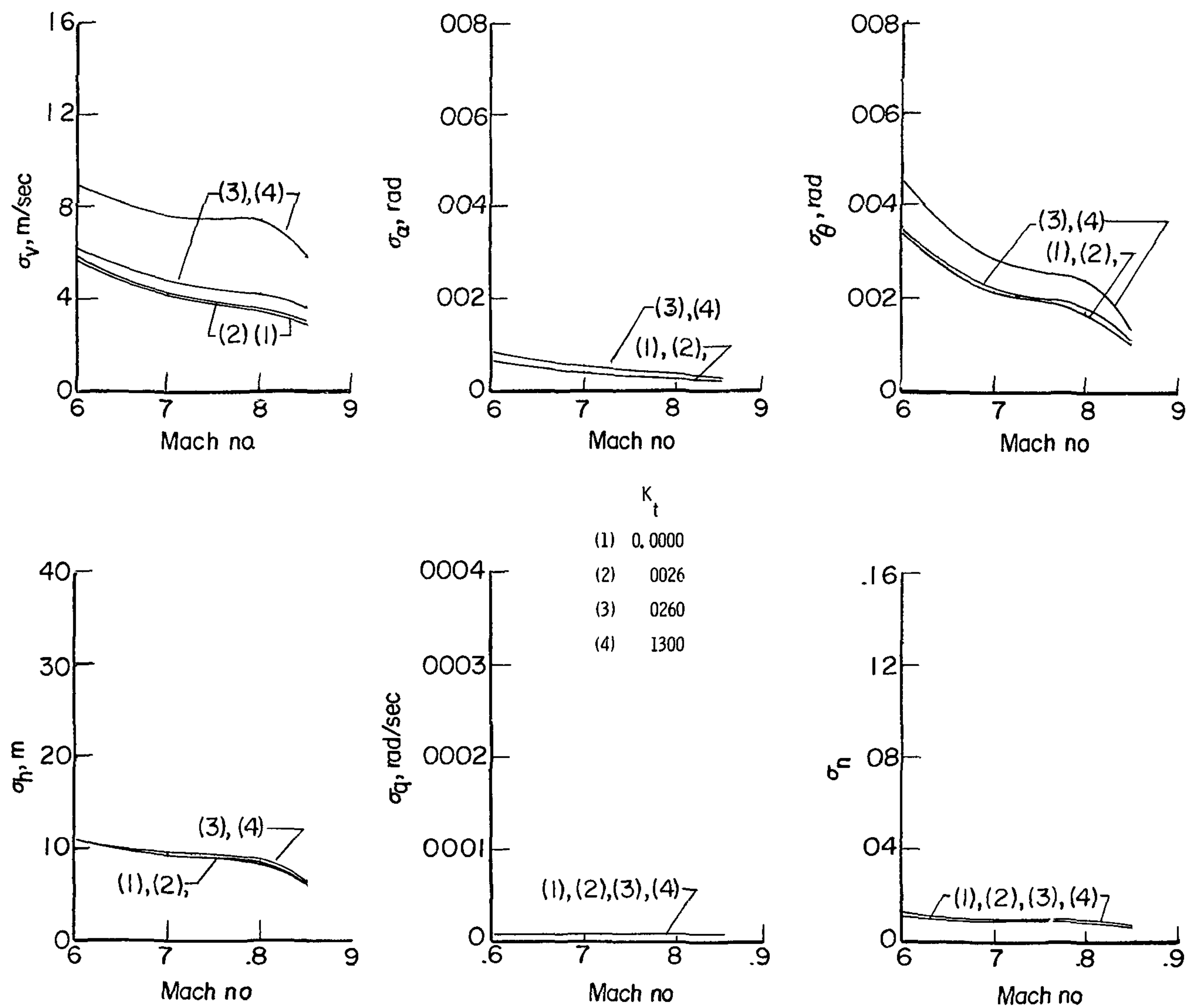
(b) Altitude = 10.668 km.

Figure 9.- Concluded.



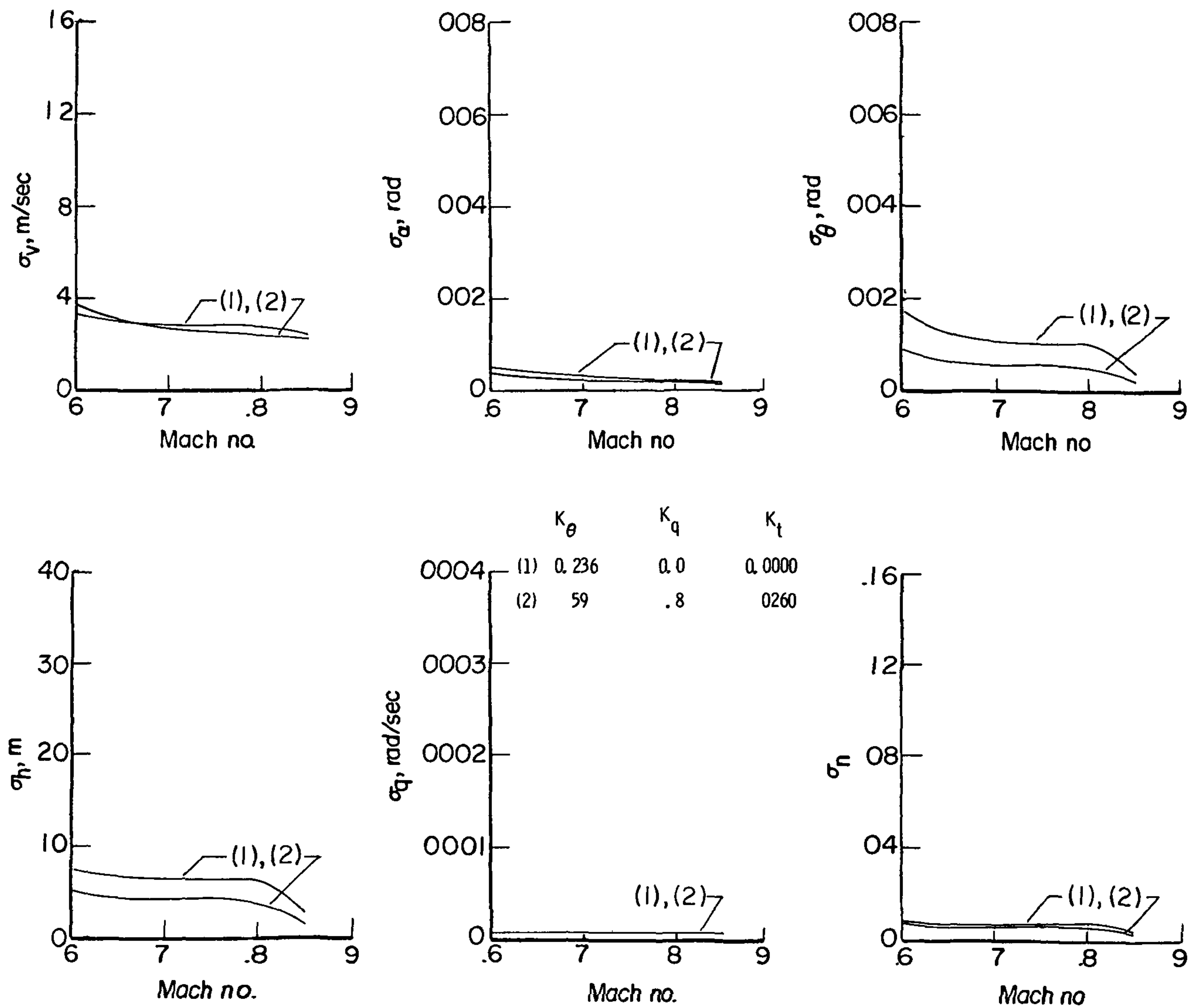
(a) Altitude = 6.096 km.

Figure 10.- Effect of autothrottle on responses of airplane to horizontal gusts. σ_u = 1 m/sec.



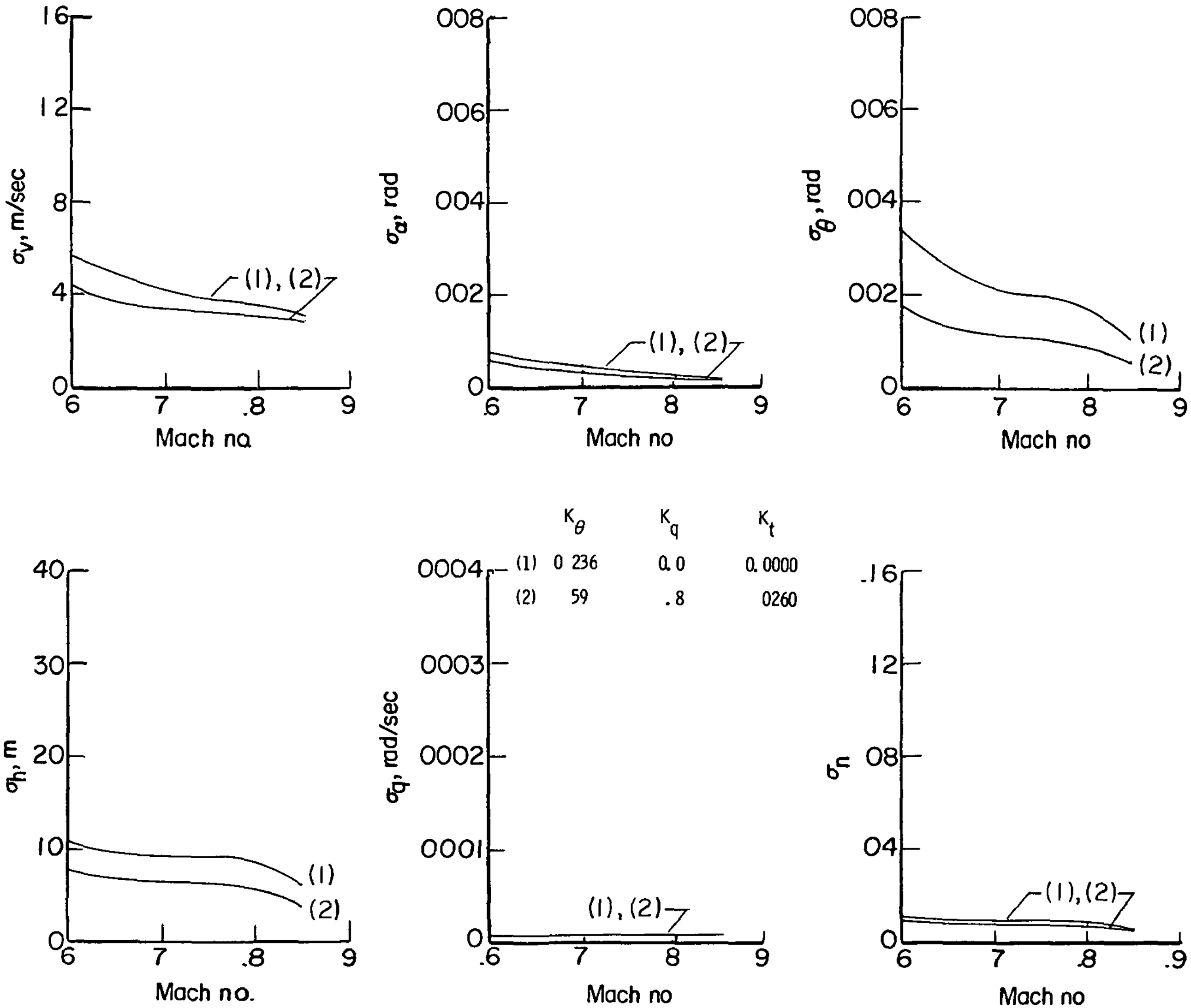
(b) Altitude = 10.668 km.

Figure 10.- Concluded.



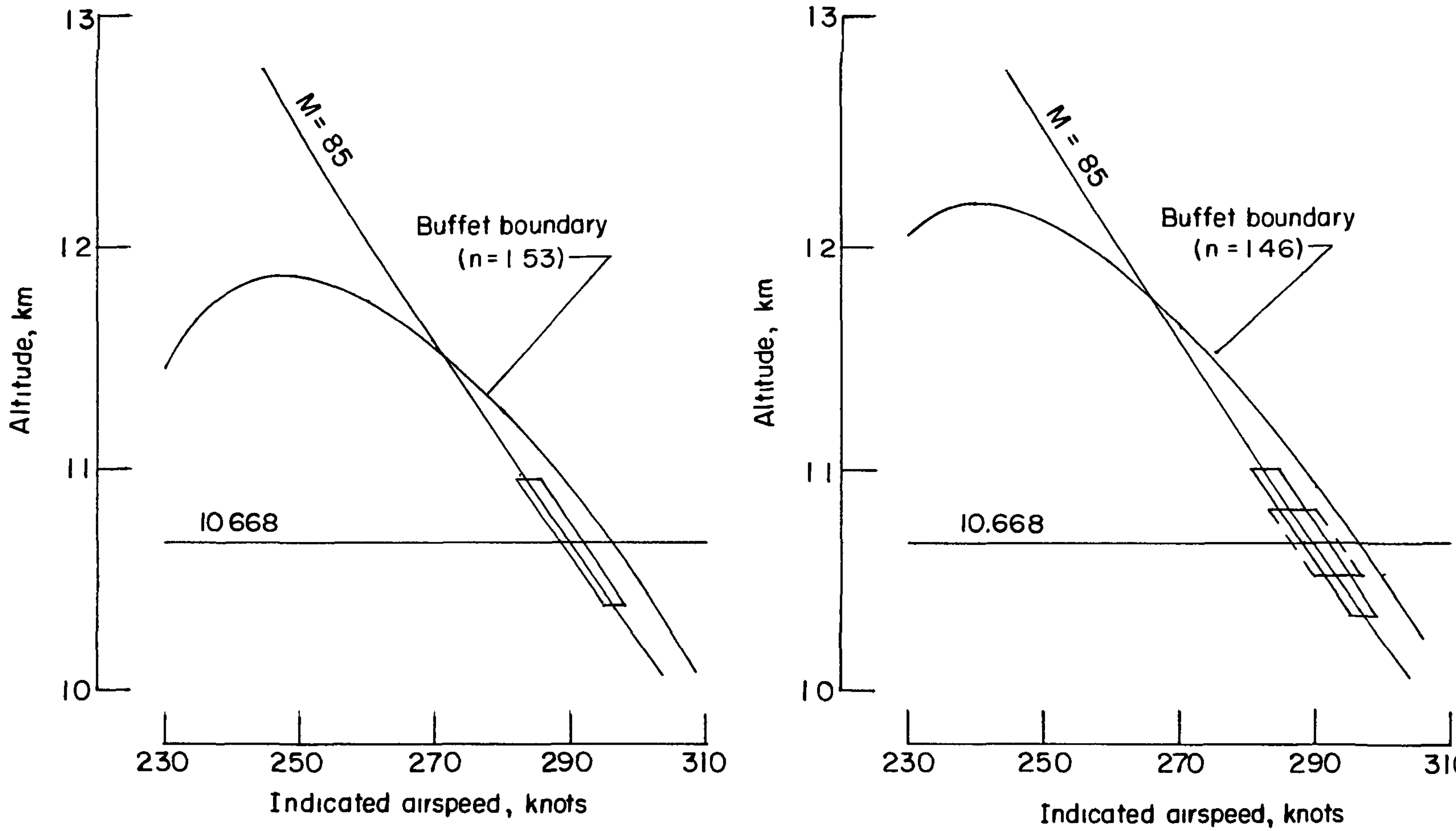
(a) Altitude = 6.096 km.

Figure 11.- Effect of combined pitch autopilot and autothrottle on responses of airplane to horizontal gusts. σ_u = 1 m/sec.



(b) Altitude = 10.668 km.

Figure 11.- Concluded.



(a) Basic airplane. $K_\theta = 0.236$; $K_q = 0.0$.

(b) Airplane with automatic controls. $K_\theta = 0.59$; $K_q = 0.8$; $K_t = 0.0260$.

Figure 12.- Proximity to buffet boundary during flight in turbulence (vertical gusts). $\sigma_w = 9$ m/sec.

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—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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